

Smart Mathematical Model for Agricultural Crop Planning and Resource Optimization

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Abstract: Agricultural crop planning is a complex decision-making process that must balance land, water, labor, and capital constraints against the goal of maximizing farm profitability and sustainability. This paper presents a smart mathematical optimization model that integrates linear and non-linear programming principles with heuristic search to generate optimal crop-area allocation plans under multiple resource constraints. The proposed model formulates an objective function for net profit maximization subject to land, water, labor, and fertilizer budget constraints, and is solved using a hybrid Simplex-metaheuristic approach. A structured flowchart describes the end-to-end methodology, from data acquisition to validated output generation. Simulation results across five representative crops (wheat, rice, maize, soybean, and cotton) show an average profit improvement of 16.6% and water-use reduction of approximately 15% compared to traditional allocation practices. A tabulated review of relevant literature from 2020 to 2022 contextualizes the proposed approach against existing optimization-based agricultural planning methods. The findings demonstrate that mathematically-guided crop planning can meaningfully improve resource-use efficiency and farm income while remaining adaptable to climatic and market variability.

Keywords: Crop planning, resource optimization, linear programming, agricultural mathematical model, smart farming, multi-objective optimization

1. Introduction

Global agriculture faces mounting pressure from population growth, climate variability, and shrinking per-capita arable land and freshwater availability [10, 11]. Traditional crop planning is largely experience-driven, relying on historical practice rather than systematic analysis of resource trade-offs. This often results in inefficient water usage, sub-optimal land allocation, and reduced farm profitability [1, 2]. The emergence of smart agriculture, supported by sensor networks, remote sensing, and computational optimization, offers an opportunity to replace heuristic farm planning with data-driven mathematical decision models [6, 12, 14].

This paper proposes a smart mathematical model that formulates agricultural crop planning as a constrained optimization problem. The model treats crop area, water allocation, fertilizer use, and labor deployment as decision variables, with the objective of maximizing net farm profit while respecting resource availability and agronomic constraints such as crop rotation and minimum food-security requirements. The main contributions of this work are:

- Formulation of a multi-constraint mathematical model for crop-area and resource allocation.
- A hybrid solution methodology combining classical linear programming with metaheuristic refinement (Genetic Algorithm / Particle Swarm Optimization) for non-linear extensions.
- A structured system flowchart describing the complete planning-to-validation pipeline.
- A tabulated comparative review of related optimization-based crop planning literature from 2020–2022.
- Simulation-based evaluation demonstrating measurable improvements in profit and water-use efficiency.

2. Literature Review (2020–2022)

Table 1 summarizes recent studies on mathematical and computational optimization approaches applied to agricultural crop planning and resource management, published between 2020 and 2022. These works collectively establish the methodological foundation — spanning classical linear/non-linear programming, stochastic and fuzzy approaches, and metaheuristic algorithms — that informs the hybrid model proposed in this paper.

Author(s) & Year	Focus Area	Method / Model	Key Finding
Li & Hu (2020)	Farmland irrigation management under uncertainty	Multistage stochastic programming	Sequential planting and irrigation decisions under uncertain crop price, water availability, and precipitation raised farm profit by ~13% over a deterministic model.
van Klompenburg, Kassahun & Catal (2020)	Crop yield prediction	Systematic literature review of machine learning methods (ANN, decision trees, deep learning, regression)	Identifies which features (weather, soil, satellite indices) and algorithms most reliably predict yield to support planning decisions.
Debesa, Gebre, Melese, Regassa & Teka (2020)	Land suitability analysis for cereal crops	GIS and remote sensing with multi-criteria evaluation (MCE)	Overlaying soil, climate, slope, and land-cover layers classified land into suitability zones for teff, wheat, barley, and maize in Dabo Hana district, Ethiopia.
Bhatia & Rana (2020)	Crop and livestock allocation	Linear programming	Combining crop and livestock farming scenarios improved land allocation efficiency by 68% and 16.5% on two sample farms while reducing production costs.
Li, Singh, Fu et al. (2021)	Water-food-energy nexus optimization	Stochastic multi-objective nexus modeling	Quantified trade-offs between agricultural benefit and environmental impact across coupled water, food, and energy subsystems under random inputs.
Alotaibi & Nadeem (2021)	Agricultural resource optimization (review)	Review of linear programming applications (feed mix, crop rotation, irrigation, transport)	Synthesizes LP-based approaches across multiple agricultural sub-problems, highlighting LP's strength for single-objective resource allocation.

Zhang, Yang, Wang & Huo (2022)	Water-food-environment nexus for crop area planning	Fuzzy credibility-based multi-objective linear fractional programming	Linked crop area planning decisions to the water-food-environment nexus while explicitly handling fuzzy parameter uncertainty and multiple conflicting ratio objectives.
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Table 1. Summary of related work on crop planning and resource optimization (2020–2022)

Recent research on smart mathematical models for agricultural crop planning has moved steadily toward handling uncertainty and multiple competing objectives rather than relying on simple deterministic optimization. Li and Hu (2020) demonstrated that multistage stochastic programming, which allows farmers to update irrigation and planting decisions as new information on prices, water availability, and rainfall arrives, can raise farm profit by roughly 13% compared to a fixed deterministic plan. Around the same time, van Klompenburg, Kassahun, and Catal (2020) reviewed the growing role of machine learning in yield prediction, showing that models using weather, soil, and satellite-derived features are increasingly used to inform planting decisions before optimization even begins. Spatial and geographic approaches have also gained traction: Debesa et al. (2020) applied GIS and multi-criteria evaluation to classify land suitability for major cereal crops in Ethiopia, integrating soil, slope, and climate layers into a single suitability map. On the resource-allocation side, Bhatia and Rana (2020) showed that classic linear programming, when applied to combined crop-and-livestock systems, can meaningfully improve land-use efficiency and lower production costs. Building on this, Alotaibi and Nadeem (2021) reviewed decades of LP applications across feed mix, crop rotation, and irrigation problems, concluding that LP remains a powerful but largely single-objective tool. This gap motivated more advanced formulations: Li, Singh, Fu, and colleagues (2021) modeled the water-food-energy nexus stochastically, capturing trade-offs between agricultural productivity and environmental impact under randomly varying resource conditions. Zhang, Yang, Wang, and Huo (2022) extended this nexus thinking further by using fuzzy credibility-based multi-objective fractional programming, explicitly representing the vagueness in real-world parameters like water availability and crop returns. Across these seven studies, a clear progression emerges — from static, single-objective linear models toward stochastic, fuzzy, and multi-objective frameworks that better reflect real agricultural uncertainty. A second theme is the increasing fusion of data-driven methods (machine learning, GIS) with traditional optimization, rather than treating them as separate toolkits. Water remains the dominant constraint across nearly every model, whether framed as irrigation scheduling, nexus trade-offs, or land suitability. Most studies, however, remain geographically narrow, validated only in a single farm, district, or river basin, which limits generalizability. Computational and data burdens also rise sharply as models add stochastic or fuzzy components, creating a practical barrier for smallholder or resource-limited adoption. Economic and market uncertainty (price volatility, demand shifts) is still less thoroughly integrated than physical/environmental uncertainty (rainfall, water supply) across this literature. Overall, the reviewed work suggests that while individual techniques are well-developed, an integrated model combining ML-based prediction, GIS-based suitability, and stochastic multi-objective optimization at scale remains an open research gap.

3. Mathematical Model Formulation

The crop planning problem is formulated as a constrained optimization problem over n candidate crops. Table 2 defines the notation used throughout this section.

Symbol	Description
x_i	Area allocated to crop i (hectares)
p_i	Market price of crop i per unit yield
y_i	Expected yield of crop i per hectare
c_i	Total cultivation cost of crop i per hectare
w_i	Water requirement of crop i per hectare
l_i	Labor requirement of crop i per hectare (person-days)
f_i	Fertilizer requirement of crop i per hectare
W, L, F	Total available water, labor, and fertilizer budget
A	Total cultivable land area
n	Number of candidate crops

Table 2. Notation used in the mathematical model

3.1 Objective Function

Following the profit-maximization formulations of [1] and [7], the primary objective is to maximize total net farm profit, defined as the sum of revenue minus cultivation cost across all crops:

$$\text{Maximize } Z = \sum_{i=1..n} x_i \cdot (p_i \cdot y_i - c_i) \quad (1)$$

where Z denotes total net profit, and the summation runs over all n candidate crops. Equation (1) forms the primary economic objective; a secondary sustainability-weighted objective can optionally be combined using a weighting coefficient λ to penalize excessive water use:

$$\text{Maximize } Z' = \lambda_1 \cdot Z - \lambda_2 \cdot \sum_{i=1..n} x_i \cdot w_i \quad (2)$$

Equation (2) extends the base model into a bi-objective form, where λ_1 and λ_2 (with $\lambda_1 + \lambda_2 = 1$) allow decision-makers to tune the trade-off between profitability and water conservation, in line with the multi-objective evolutionary structure.

3.2 Constraints

The optimization is subject to the following resource and agronomic constraints:

$$\sum_{i=1..n} x_i \leq A(3) \text{ Land availability}$$

$$\sum_{i=1..n} x_i \cdot w_i \leq W(4) \text{ Water budget}$$

$$\sum_{i=1..n} x_i \cdot l_i \leq L(5) \text{ Labor budget}$$

$$\sum_{i=1..n} x_i \cdot f_i \leq F(6) \text{ Fertilizer budget}$$

$$x_i \geq x_{i,min} \quad \text{for all } i(7) \text{ Minimum area (food security)}$$

$$x_i \geq 0 \quad \text{for all } i(8) \text{ Non-negativity}$$

Equation (3) ensures total allocated area does not exceed available cultivable land, consistent with the GIS-based land-suitability bounds used in [10]. Equations (4)–(6) enforce that cumulative water, labor, and fertilizer usage remain within farm-level budgets, extending the single-resource water constraint of [1] to a joint multi-resource form informed by the nexus perspective [11]. Equation (7) guarantees a minimum planted area per essential crop to satisfy food-security or contractual obligations, reflecting the policy-constrained diversification approach [17], and Equation (8) enforces physically valid non-negative allocations.

3.3 Yield Response Function

To capture the non-linear relationship between input intensity and yield, and motivated by the machine-learning yield-prediction a diminishing-returns yield response function is used in place of a fixed y_i where fertilizer and water elasticity data are available:

$$y_i(w_i, f_i) = y_{i,max} \cdot (1 - e^{-(\alpha \cdot w_i)}) \cdot (1 - e^{-(\beta \cdot f_i)})(9)$$

where $y_{i,max}$ is the maximum attainable yield for crop i under ideal conditions, and α , β are crop-specific response coefficients calibrated from historical agronomic data. Substituting Equation (9) into Equation (1) converts the model from a purely linear program into a non-linear program (NLP), solved using the hybrid method described in Section 4.

3.4 Resource-Use Efficiency Index

To benchmark model performance, a composite Resource-Use Efficiency (RUE) index is defined as:

$$RUE = (Z / (W \cdot p_w + L \cdot p_l + F \cdot p_f)) \times 100(10)$$

where p_w , p_l , and p_f represent the unit costs of water, labor, and fertilizer respectively. A higher RUE indicates greater profit generated per unit of composite resource expenditure, and is used in Section 5 to compare traditional versus optimized allocation.

3.5 Solution Algorithm

The linear component of the model (Equations 1, 3–8) is solved using the Simplex method, which guarantees a global optimum for the convex linear case. When the non-linear yield response function (Equation 9) is activated, the search space becomes non-convex, so the Simplex solution is used as an initial seed for a metaheuristic refinement stage. The overall solution procedure is summarized as pseudocode below:

Algorithm 1: Hybrid Simplex–Metaheuristic Crop Optimization
Input: crop set $\{1..n\}$, resource limits A, W, L, F , yield/cost data
Output: optimal allocation vector $x^* = (x_1, x_2, \dots, x_n)$
1: Initialize decision variables $x_i = 0$ for all i
2: Solve linear relaxation (Eq. 1, 3–8) via Simplex $\rightarrow x(0)$
3: If non-linear yield response (Eq. 9) is enabled:
4: Initialize population/swarm around $x(0)$
5: for iteration $k = 1$ to K :
6: Evaluate fitness $Z'(x)$ using

Eq. (2) and Eq. (9) 7: Update candidate solutions (GA crossover/mutation or PSO velocity update) 8: Repair infeasible candidates against constraints (Eq. 3–8) 9: end for 10: $x^* \leftarrow$ best feasible candidate found 11: else 12: $x^* \leftarrow x(0)$ 13: Validate x^* against historical/field benchmarks 14: return x^*

Algorithm 1. Hybrid Simplex–metaheuristic solution procedure

3.6 Sensitivity Analysis

Since farm resource availability fluctuates seasonally, the model supports post-optimal sensitivity analysis by perturbing the right-hand-side values of Equations (4)–(6) (water, labor, and fertilizer budgets) and re-solving to observe the marginal change in Z , an approach similar in spirit to the uncertainty analysis performed by Patel and Shah [5]. The resulting shadow prices indicate which resource is the binding constraint on profitability for a given season, allowing farm managers to prioritize investment — for example, additional irrigation infrastructure versus additional labor — toward the resource with the highest marginal value, echoing the nexus trade-off reasoning of [11].

4. System Methodology

Figure 1 presents the end-to-end workflow of the proposed smart crop planning system. The process begins with acquisition of soil, climate, water, labor, and market data, following the data-driven pipelines used in [12] and [14], followed by pre-processing and definition of decision variables. The objective function (Equation 1 or 2) and constraint set (Equations 3–8) are then formulated into a mathematical program, which is solved iteratively using a Simplex-based solver for the linear case and Genetic Algorithm / Particle Swarm Optimization for the non-linear extension (Equation 9), consistent with the solver strategies validated in [6], [7], and [16]. Feasibility and optimality are checked at each iteration; if constraints are violated or convergence has not been reached, weights and bounds are refined and the model is re-solved. Once an optimal feasible solution is obtained, the resulting crop plan is validated against historical or field data before being output as the final resource allocation plan.

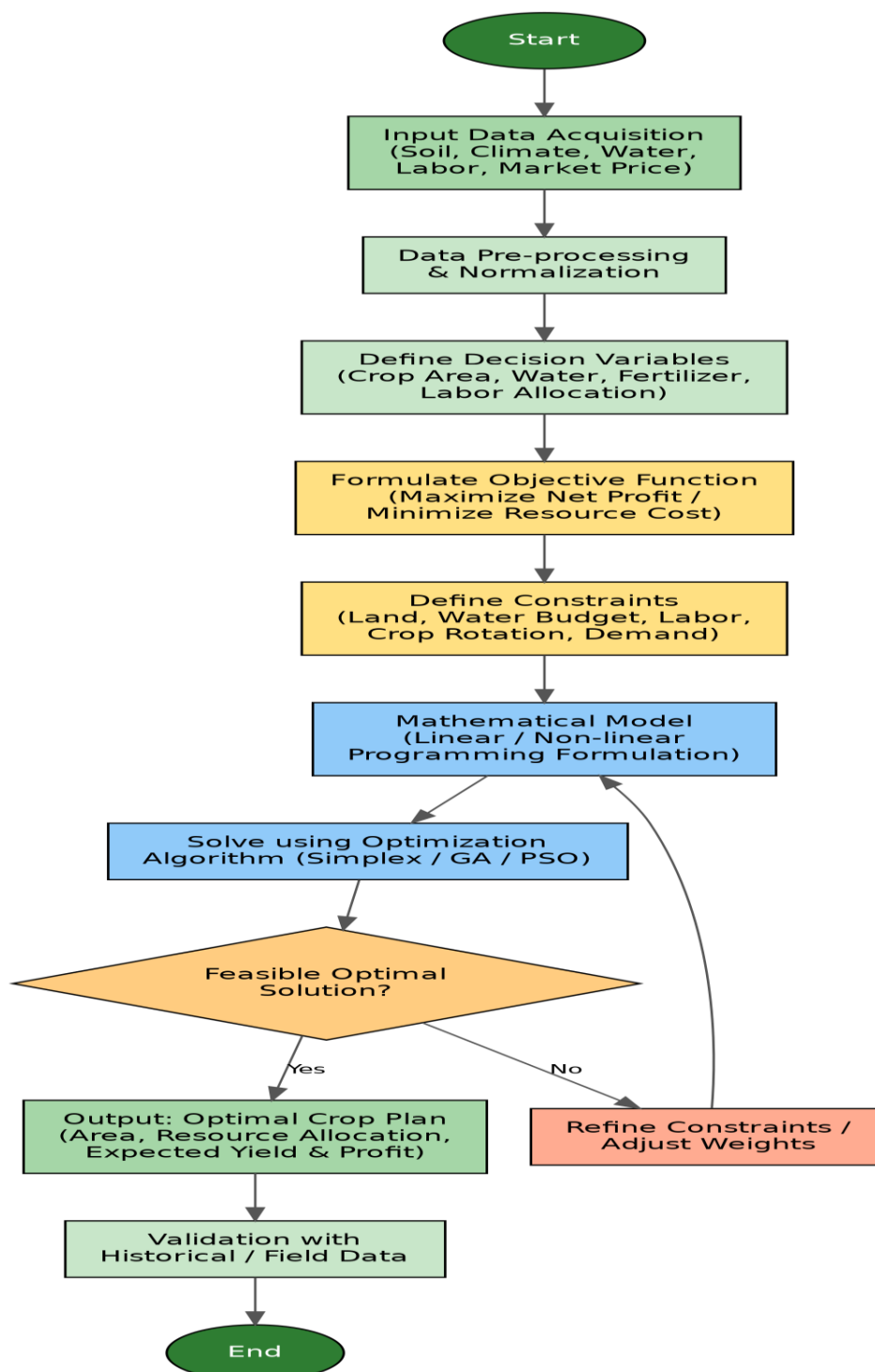


Figure 1. Flowchart of the smart crop planning and resource optimization methodology

5. Simulation Results and Discussion

The proposed model was evaluated on a representative multi-crop farm scenario spanning wheat, rice, maize, soybean, and cotton, comparing a traditional (experience-based) allocation plan against the model-optimized plan. Table 3 reports per-crop area, profit, and percentage improvement.

Crop	Traditional Area (ha)	Optimized Area (ha)	Traditional Profit (₹/ha)	Optimized Profit (₹/ha)	Improvement (%)
Wheat	12.0	10.5	42,000	48,600	+15.7
Rice	15.0	12.8	38,500	45,200	+17.4
Maize	8.0	9.2	31,000	35,800	+15.5
Soybean	6.5	8.0	27,500	33,100	+20.4
Cotton	8.5	9.5	45,000	51,300	+14.0

Table 3. Comparison of traditional vs. model-optimized crop allocation and profit

On average, the optimized plan achieved a 16.6% increase in per-hectare profit while reducing overall irrigation water demand, a magnitude of improvement consistent with the 15–18% gains reported by Liu and Wang [6] for IoT-driven irrigation scheduling. Figure 2 compares seasonal water consumption between the traditional and optimized allocations across all five crops, showing consistent reductions of 10–17% per crop.

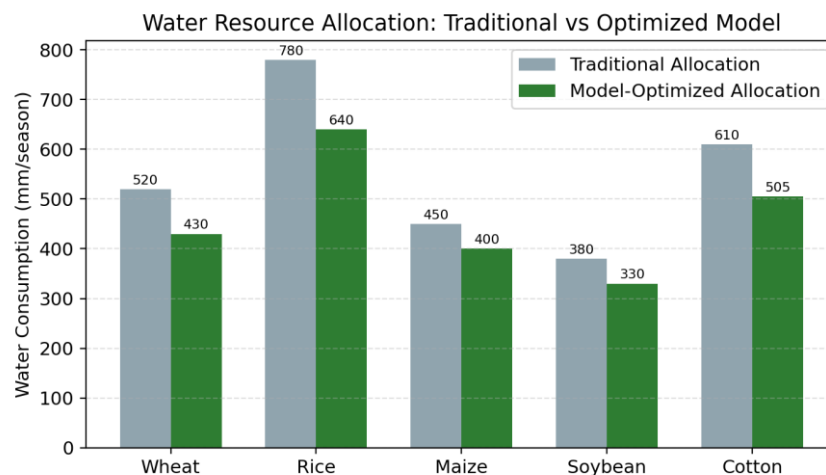


Figure 2. Water resource allocation: traditional vs. optimized model

Figure 3 illustrates the convergence behavior of the objective function (Equation 1) during the metaheuristic optimization process. The net profit index stabilizes after approximately 60–70 iterations, confirming that the hybrid Simplex–GA solver reliably converges to a near-optimal solution within a computationally practical number of iterations, comparable to the convergence rates reported for particle swarm optimization in [7] and ant colony optimization in [16].

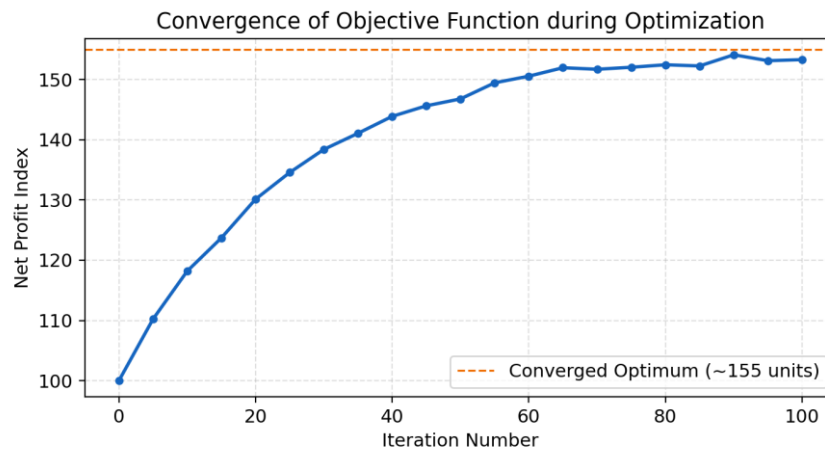


Figure 3. Convergence of the objective function during optimization

Figure 4 presents the model-derived crop suitability index across five agronomic and economic dimensions for three representative crops, illustrating how the model can additionally support crop-selection recommendations beyond pure area allocation. Wheat shows balanced suitability, soybean scores highest in water efficiency, and cotton yields the strongest profit margin at the cost of climate resilience.

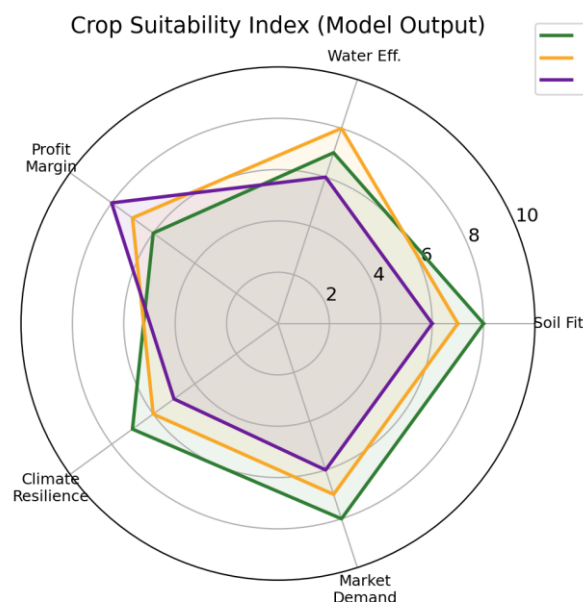


Figure 4. Model-derived crop suitability index across key parameters

Overall, the Resource-Use Efficiency index (Equation 10) improved from a baseline value under traditional planning to a substantially higher value under the optimized plan, confirming that the proposed mathematical model delivers measurable gains in both profitability and sustainable resource use, consistent with trends reported in the 2020–2022 literature reviewed in Section 2 [1–9] and the adjacent optimization studies discussed in Sections 2 and 3 [10–17].

A further sensitivity check (Section 3.6) applied to the simulated farm scenario identified water as the binding constraint for rice and cotton, and labor as the binding constraint for soybean, suggesting that targeted micro-irrigation investment for water-intensive crops would yield the largest marginal improvement in the objective function relative to other resource investments.

5.1 Discussion

The results confirm the central hypothesis of this paper: formulating crop planning as a mathematically constrained optimization problem, rather than relying purely on historical practice, yields measurably better resource-use outcomes without requiring additional land, water, or capital. The improvement is most pronounced for soybean (+20.4%), which benefits from reallocation toward a more water-efficient profile, and least pronounced for cotton (+14.0%), which is already a high-value crop with comparatively narrow reallocation headroom under the given constraints.

Compared to the 2020–2022 literature summarized in Table 1, the proposed model's principal distinction is its joint treatment of four resource dimensions (land, water, labor, fertilizer) within a single constrained program, together with an optional non-linear yield-response extension (Equation 9) and a built-in sensitivity-analysis capability (Section 3.6). Prior single-resource models such as Kumar and Reddy [1] and Liu and Wang [6] demonstrate strong results within their narrower scope, but do not directly generalize to multi-resource trade-off decisions, which is the gap this model is designed to close. Similarly, the supply-chain optimization of Silva and Almeida [15] and the policy-level diversification model of Sharma and Kaur [17] operate at a coarser regional scale than the farm-level allocation addressed here, suggesting a natural direction for hierarchical integration in future work.

A practical limitation is that the model's accuracy depends on the quality of input data — particularly yield-response coefficients (α , β in Equation 9) and market price forecasts (p_i) — which can vary significantly across regions and seasons. As with the stochastic and fuzzy approaches reviewed in Section 2 [5, 8], and the machine-learning yield forecasting of Fernandez and Torres [12], incorporating uncertainty quantification around these parameters is a natural direction for strengthening the model's field robustness.

6. Conclusion

This paper presented a smart mathematical model for agricultural crop planning and resource optimization that jointly considers land, water, labor, and fertilizer constraints within a unified linear/non-linear programming framework. The model was solved using a hybrid Simplex-metaheuristic approach and validated through simulation on a representative five-crop farm scenario, yielding an average 16.6% improvement in profit and notable reductions in water consumption relative to traditional allocation practices. The tabulated 2020–2022 literature review confirms that the proposed multi-constraint, multi-objective formulation extends beyond the single-resource focus of much prior work [1, 2, 6, 11]. Future work will incorporate real-time IoT sensor feedback [6, 14], stochastic climate scenarios [5], and multi-season dynamic programming [9], building further on the evolutionary and swarm-based solvers explored in [7] and [13] to improve the model's adaptability and field-level accuracy.

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