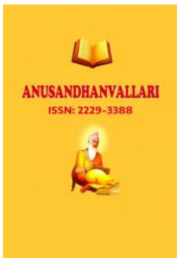

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Fixed Point Results for Hybrid Multi-Valued Mappings in Metric Spaces

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Abstract: The purpose of the present paper is to investigate fixed point results for a class of hybrid multi-valued mappings defined on complete metric spaces. The contractive condition considered in the study combines the ordinary distance between two points with the distances of the points from their respective images. In this sense, the condition incorporates features of both Nadler-type and Kannan-type contractions. By using the Hausdorff metric and an iterative selection procedure, a fixed point existence theorem is established for multi-valued mappings taking values in the family of nonempty closed and bounded subsets of a complete metric space. Several immediate consequences are derived, including the classical multi-valued contraction case and a Kannan-type multi-valued fixed point result. The corresponding single-valued version is also obtained, for which uniqueness of the fixed point follows. Two illustrative examples are presented, supported by explanatory figures. In particular, a finite metric-space example shows that the proposed hybrid condition may hold even when the standard Nadler contraction condition is not satisfied.

Keywords: Fixed point; multi-valued mapping; metric space; Hausdorff metric; hybrid contraction; Kannan-type contraction; Nadler contraction.

2020 Mathematics Subject Classification: 47H10, 54H25.

1. Introduction

Fixed point theory occupies an important position in nonlinear analysis and has applications in functional equations, differential equations, integral equations, optimization, control theory and mathematical economics. The classical contraction principle of Banach provides one of the fundamental existence and uniqueness results in metric fixed point theory. Banach established that every contraction on a complete metric space possesses a unique fixed point.

The development of multi-valued analysis created the need for fixed point principles in which a point is mapped not to a single point but to a nonempty subset of the underlying space. Nadler extended the contraction principle to multi-valued mappings by employing the Hausdorff metric. Specifically, for a complete metric space (X, d), a multi-valued mapping F : X → CB(X) satisfying

$$H(F(x), F(y)) \leq \lambda\, d(x, y), \qquad 0 \leq \lambda < 1$$

has at least one fixed point. Here, CB(X) denotes the family of all nonempty closed and bounded subsets of X, while H denotes the Hausdorff metric induced by d.

Subsequently, fixed point theory was extended through contractive conditions different from the ordinary Banach contraction. Kannan introduced a contraction based on distances between points and their respective images, while Reich studied broader classes of contractive mappings. Mizoguchi and Takahashi later developed an important generalization of Nadler-type multivalued contractions on complete metric spaces. Bhimanatham and Iyer (2025) Further developments have considered set-valued contractions, generalized metric structures, graph contractions and stability properties. Recent research also shows that multi-valued fixed point theory continues to be extended to nonlinear graph contractions and related metric structures.

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Motivated by these developments, the present study considers a contractive condition that simultaneously involves the distance d(x, y) and the point-to-image distances D(x, F(x)) and D(y, F(y)). Throughout this paper, such a condition is referred to as a hybrid multi-valued contractive condition. The principal objective is to establish the existence of a fixed point under this blended contraction and to identify classical results that arise as special cases. Figure 1 (Section 2) gives a visual summary of how these three distances interact.

2. Preliminaries

We begin with definitions required for the development of the main results.

Definition 2.1. Metric Space

Let X be a nonempty set. A function

$$d : X \times X \to [0, \infty)$$

is called a metric on X if for all x, y, z ∈ X:

$$\begin{aligned} &d(x,y) = 0 \iff x = y, \\ &d(x,y) = d(y,x), \\ &d(x,z) \le d(x,y) + d(y,z) \end{aligned}$$

The pair (X, d) is then called a metric space.

Definition 2.2. Complete Metric Space

A metric space (X, d) is said to be complete if every Cauchy sequence in X converges to an element of X.

Definition 2.3. Family CB(X)

Let

$$CB(X) = \{A \subseteq X : A \neq \varnothing,\ A \text{ is closed and bounded}\}$$

Definition 2.4. Distance from a Point to a Set

For x ∈ X and a nonempty subset A ⊆ X, define

$$D(x, A) = \inf\{d(x,a) : a \in A\}$$

Clearly,

$$D(x, A) = 0$$

whenever x ∈ A. If A is closed, the converse also holds.

Definition 2.5. Hausdorff Metric

For A, B ∈ CB(X), define

$$H(A,B) = \max\{\sup_{a \in A} D(a,B),\ \sup_{b \in B} D(b,A)\}$$

The function H is the Hausdorff metric induced by d.

Definition 2.6. Multi-Valued Mapping

A mapping

$$F : X \to CB(X)$$

is called a multi-valued or set-valued mapping.

Definition 2.7. Fixed Point

A point x* ∈ X is called a fixed point of a multi-valued mapping F if

$$x^* \in F(x^*)$$

The set of all fixed points of F is denoted by

$$Fix(F) = \{x \in X : x \in F(x)\}$$

Definition 2.8. Strict Fixed Point or Endpoint

A point x* ∈ X is called a strict fixed point of F if

$$F(x^*) = \{x^*\}$$

Definition 2.9. Hybrid Multi-Valued Contraction

Let (X, d) be a metric space and

$$F : X \to CB(X)$$

For the purposes of the present paper, F will be called a hybrid multi-valued contraction if there exist constants satisfying

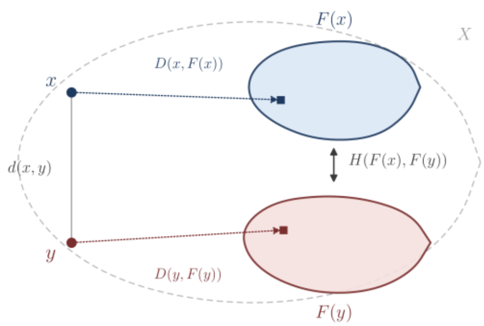
$$a, b \geq 0 \quad \text{satisfying} \quad a + 2b < 1$$

such that

$$H(F(x), F(y)) \ \leq \ a\, d(x, y) \ + \ b\big[D(x, F(x)) + D(y, F(y))\big] \tag{2.1}$$

for every x, y ∈ X.

The condition is hybrid because the right-hand side combines an ordinary interpoint distance with two point-to-image distances. Figure 1 illustrates the geometric meaning of condition (2.1): the Hausdorff distance between the two image sets F(x) and F(y) is bounded jointly by the interpoint distance d(x, y) and the two point-to-image distances D(x, F(x)) and D(y, F(y)).



3. Main Fixed Point Result

Theorem 3.1

Let (X, d) be a complete metric space and let

$$F : X \to CB(X)$$

satisfy

$$H(F(x), F(y)) \leq a\, d(x, y) + b[D(x, F(x)) + D(y, F(y))] \tag{3.1}$$

for all x, y ∈ X, where a, b ≥ 0 and

$$a + 2b < 1 \tag{3.2}$$

Then F possesses at least one fixed point in X.

Proof

Choose an arbitrary point x_0 ∈ X. Since $F(x_0) \neq \emptyset$, select

$$x_1 \in F(x_0)$$

If $x_1 = x_0$, then

$$x_1 = x_0 \implies x_0 \in F(x_0)$$

and hence x_0 is a fixed point.

Assume therefore that

$$x_1 \neq x_0$$

Since $x_n \in F(x_{n-1})$, for every n ≥ 1, by the definition of the Hausdorff metric we may choose

$$x_{n+1} \in F(x_n)$$

such that

$$d(x_n, x_{n+1}) \;\leq\; H(F(x_{n-1}), F(x_n)) + \varepsilon_n \tag{3.3}$$

where

$$\varepsilon_n = (1-b)2^{-n} > 0$$

Put

$$\delta_n = d(x_{n-1}, x_n), \qquad n \geq 1$$

From (3.1) and (3.3),

$$\begin{aligned}\delta_{n+1} \leq\; & a\, d(x_{n-1}, x_n) \\ & + b\big[D(x_{n-1}, F(x_{n-1})) + D(x_n, F(x_n))\big] + \varepsilon_n\end{aligned}$$

Because

$$x_n \in F(x_{n-1})$$

we have

Similarly, since $x_{n+1} \in F(x_n)$, we obtain the pair of bounds

$$\begin{aligned}D(x_{n-1}, F(x_{n-1})) &\leq d(x_{n-1}, x_n) = \delta_n, \\ D(x_n, F(x_n)) &\leq d(x_n, x_{n+1}) = \delta_{n+1}\end{aligned}$$

Hence,

$$\delta_{n+1} \ \leq \ a\delta_n + b(\delta_n + \delta_{n+1}) + (1-b)2^{-n}$$

Therefore,

$$(1-b)\delta_{n+1} \ \leq \ (a+b)\delta_n + (1-b)2^{-n}$$

Consequently,

$$\delta_{n+1} \ \leq \ q\,\delta_n + 2^{-n} \tag{3.4}$$

where

$$q = \frac{a+b}{1-b}$$

Now

$$a + 2b < 1$$

implies

$$a + b < 1 - b$$

and therefore

$$0 \leq q < 1$$

Repeated application of (3.4) gives

$$\delta_{n+1} \ \leq \ q^n\delta_1 + \sum_{j=1}^{n} q^{n-j}2^{-j} \tag{3.5}$$

Since $0 \leq q < 1$,

$$\sum_{n=1}^{\infty} q^n < \infty \qquad \text{and} \qquad \sum_{j=1}^{\infty} 2^{-j} < \infty$$

It follows from (3.5) that

$$\sum_{n=1}^{\infty} \delta_n < \infty$$

Therefore, for $m > n$,

$$d(x_n, x_m) \ \leq \ \sum_{k=n+1}^{m} \delta_k \ \longrightarrow \ 0$$

as $n \to \infty$.

Hence $\{x_n\}$ is a Cauchy sequence.

Since X is complete, there exists $u \in X$ such that

$$x_n \longrightarrow u \tag{3.6}$$

We now prove that

$$u \in F(u)$$

Since $x_{n+1} \in F(x_n)$,

$$D(x_{n+1}, F(u)) \leq H(F(x_n), F(u))$$

Therefore,

$$D(u, F(u)) \leq d(u, x_{n+1}) + D(x_{n+1}, F(u))$$
$$\leq d(u, x_{n+1}) + H(F(x_n), F(u))$$

Applying the hybrid contractive condition,

$$D(u, F(u)) \ \leq d(u, x_{n+1}) + a\, d(x_n, u) + b\big[D(x_n, F(x_n)) + D(u, F(u))\big]$$

Furthermore,

$$D(x_n, F(x_n)) \leq d(x_n, x_{n+1}) = \delta_{n+1}$$

Taking the limit as n ⟶ ∞, and using

$$x_n \to u, \qquad x_{n+1} \to u, \qquad \delta_{n+1} \to 0$$

we obtain

$$D(u, F(u)) \leq b\, D(u, F(u))$$

Thus, since b < 1,

$$(1-b)D(u, F(u)) \leq 0 \implies D(u, F(u)) = 0$$

Because F(u) is closed,

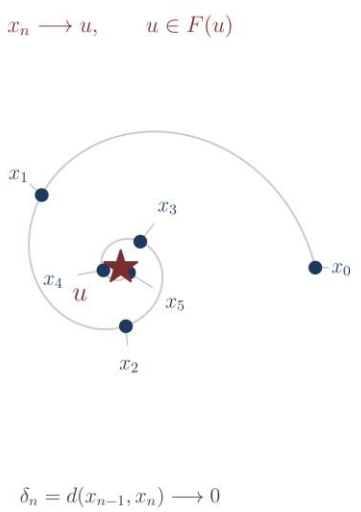
$$u \in F(u)$$

Hence u is a fixed point of F.

$$Fix(F) \neq \varnothing$$

□

Figure 2 depicts this Picard-type iterative scheme: successive terms x_n move closer together at a rate controlled by q, forcing $\{x_n\}$ to be Cauchy, and completeness of X then yields a limit u that Theorem 3.1 shows lies in its own image.



4. Consequences of the Main Theorem

Corollary 4.1: Nadler-Type Contraction

Let (X, d) be complete and

$$F : X \to CB(X)$$

satisfy

$$H(F(x), F(y)) \le a\, d(x, y)$$

for all x, y ∈ X, where

$$0 \le a < 1$$

Then F has a fixed point.

Proof

Take b = 0 in Theorem 3.1. Then

$$a + 2b = a < 1$$

Hence the conclusion follows immediately.

□

This recovers the basic form of Nadler's multi-valued contraction principle.

Corollary 4.2: Kannan-Type Multi-Valued Contraction

Let (X, d) be complete and

$$F : X \to CB(X)$$

satisfy

$$H(F(x), F(y)) \le b[D(x, F(x)) + D(y, F(y))] \tag{4.1}$$

for every x, y ∈ X, where

$$0 \le b < \tfrac{1}{2}$$

Then F has a fixed point.

Proof

Take a = 0 in Theorem 3.1. Since

$$2b < 1$$

all conditions of the theorem are satisfied.

□

Corollary 4.3: Single-Valued Hybrid Contraction

Let (X, d) be a complete metric space and

$$f : X \to X$$

satisfy

$$d(f(x), f(y)) \leq a\, d(x,y) + b[d(x, f(x)) + d(y, f(y))] \tag{4.2}$$

for all x, y ∈ X, where

$$a, b \geq 0, \qquad a + 2b < 1$$

Then f has a unique fixed point.

Proof

Define

$$F(x) = \{f(x)\}$$

Then

$$H(F(x), F(y)) = d(f(x), f(y))$$

and

$$D(x, F(x)) = d(x, f(x))$$

Thus Theorem 3.1 ensures existence of a fixed point.

Suppose u and v are two fixed points. Then

$$f(u) = u, \qquad f(v) = v$$

Therefore,

$$\begin{aligned} d(u,v) &= d(f(u), f(v)) \\ &\leq a\, d(u,v) + b[d(u, f(u)) + d(v, f(v))] \\ &= a\, d(u,v) \end{aligned}$$

Hence, since a < 1,

$$(1-a)d(u,v) \leq 0 \implies d(u,v) = 0 \implies u = v$$

Therefore the fixed point is unique.

□

5. Strict Fixed Points

Proposition 5.1

Under the assumptions of Theorem 3.1, the mapping F can have at most one strict fixed point.

Proof

Suppose u and v are strict fixed points. Then

$$F(u) = \{u\}, \qquad F(v) = \{v\}$$

Therefore,

$$H(F(u), F(v)) = d(u,v)$$

Since

$$D(u, F(u)) = D(v, F(v)) = 0$$

condition (3.1) gives

$$d(u,v) \le a\, d(u,v)$$

Because a < 1,

$$(1-a)d(u,v) \le 0 \implies u = v$$

Thus, if a strict fixed point exists, it is unique.

□

It is important to distinguish this result from ordinary fixed-point uniqueness. In general, a multi-valued contraction may possess more than one ordinary fixed point, so Theorem 3.1 deliberately claims existence, not uniqueness.

6. Illustrative Examples

Example 6.1

Consider

$$X = [0,1]$$

with the usual metric

$$d(x,y) = |x-y|$$

Define

$$F : X \to CB(X)$$

by

$$F(x) = \left[0, \tfrac{x}{3}\right]$$

For x, y ∈ X,

$$H(F(x),F(y)) = \left|\tfrac{x}{3} - \tfrac{y}{3}\right| = \tfrac{1}{3}|x-y|$$

Hence,

$$H(F(x),F(y)) \le \tfrac{1}{3} d(x,y)$$

Thus Theorem 3.1 applies with

$$a = \tfrac{1}{3},\ b = 0 \qquad \implies \qquad a + 2b = \tfrac{1}{3} < 1$$

F has a fixed point.

To determine it, suppose

$$x \in F(x)$$

Then

$$x \in \left[0, \tfrac{x}{3}\right]$$

For x ≥ 0, this requires

$$x \leq \frac{x}{3} \implies x = 0$$

Therefore,

$$Fix(F) = \{0\}$$

7. An Example Beyond the Standard Nadler Contraction

The following example is particularly useful because the hybrid theorem applies while the usual Nadler contraction with constant strictly less than one does not.

Example 7.1

Let

$$X = \{0, 1, 10, 11\}$$

with the usual metric inherited from $\mathbb{R}$:

$$d(x, y) = |x - y|$$

Because X is finite, it is complete.

Define

$$F : X \to CB(X)$$

by

$$F(0) = \{10\}, \quad F(1) = \{10, 11\},$$
$$F(10) = \{10\}, \ F(11) = \{10\}$$

First calculate the point-to-image distances:

$$D(0, F(0)) = 10, \quad D(1, F(1)) = 9, \quad D(10, F(10)) = 0, \quad D(11, F(11)) = 1$$

Take

$$a = 0, \ b = \frac{1}{9} \qquad \implies \qquad a + 2b = \frac{2}{9} < 1$$

For the distinct pairs of points, we obtain:

x, y	H(F(x),F(y))	D(x,F(x))+D(y,F(y))	(1/9)[D(x,F(x))+D(y,F(y))]
0, 1	1	19	19/9
0, 10	0	10	10/9
0, 11	0	11	11/9
1, 10	1	9	1
1, 11	1	10	10/9
10, 11	0	1	1/9

Consequently,

$$H(F(x), F(y)) \leq \frac{1}{9}[D(x, F(x)) + D(y, F(y))]$$

for all x, y ∈ X.

Thus Corollary 4.2 guarantees the existence of a fixed point.

Indeed,

$$10 \in F(10) = \{10\}$$

Therefore, in fact,

$$Fix(F) = \{10\}$$

Why the standard Nadler condition fails

For x = 0 and y = 1,

$$H(F(0), F(1)) = H(\{10\}, \{10, 11\}) = 1$$

while

$$d(0, 1) = 1$$

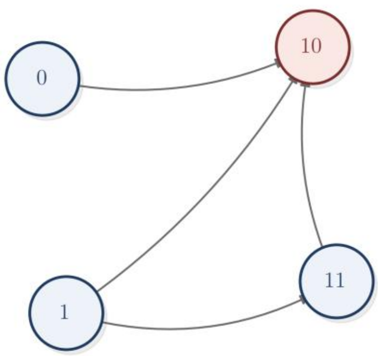
Thus

$$\frac{H(F(0),F(1))}{d(0,1)} = 1$$

Hence there cannot exist any constant λ < 1 such that H(F(x),F(y)) ≤ λ d(x,y) for all x, y ∈ X.

Therefore this mapping is not a Nadler contraction, although it satisfies the hybrid/Kannan-type condition considered in the present paper. Paul et al. (2025)

This example establishes that the class covered by Theorem 3.1 is not restricted to the ordinary Nadler contraction case. Figure 3 shows the directed graph of F on X: every point is eventually sent into the self-loop at 10, which is the unique fixed point, even though the ratio H(F(0),F(1))/d(0,1) = 1 rules out a genuine Nadler contraction.



8. Discussion

The principal result demonstrates that a fixed point can be obtained without requiring the Hausdorff distance between F(x) and F(y) to depend exclusively on d(x, y). Instead, the quantities

$$D(x, F(x)) \quad \text{and} \quad D(y, F(y))$$

also contribute to the contractive mechanism.

The condition

$$a + 2b < 1$$

has a direct mathematical role in the proof. It gives

$$q = \tfrac{a+b}{1-b} < 1$$

which provides the contraction ratio required to control the successive distances

$$d(x_n, x_{n+1})$$

Thus, the restriction on the constants is not arbitrary; it is precisely what permits the iterative sequence to become Cauchy.

The result contains several familiar situations. When

$$b = 0$$

the condition reduces to the ordinary Hausdorff contraction used in Nadler-type fixed point theory. When

$$a = 0$$

a Kannan-type set-valued condition is obtained. For singleton-valued mappings, the theorem produces a single-valued hybrid contraction and additionally guarantees uniqueness.

The finite example in Section 7 is particularly significant because it demonstrates that the presence of the point-to-image terms can compensate for the failure of a strict Hausdorff Lipschitz constant. Hence the hybrid formulation provides a mathematically meaningful extension of the basic contraction framework rather than merely rewriting the standard condition. Mukherjee et al. (2025)

9. Conclusion

This paper has considered a class of hybrid multi-valued mappings on complete metric spaces. A fixed point theorem has been established under the contractive condition

$$H(F(x), F(y)) \;\leq\; a\, d(x, y) + b[D(x, F(x)) + D(y, F(y))]$$

where

$$a, b \geq 0 \qquad \text{and} \qquad a + 2b < 1$$

An iterative selection sequence was constructed and shown to be Cauchy. Completeness of the underlying metric space then produced a limit, while the hybrid contractive condition ensured that the limit belongs to its image.

The theorem yields Nadler-type and Kannan-type multivalued contractions as immediate consequences. The corresponding single-valued result guarantees existence as well as uniqueness of the fixed point. It has also been shown that any strict fixed point of the multi-valued mapping is unique. The illustrative examples confirm the applicability of the results, and one example demonstrates that the hybrid condition may be satisfied even when the ordinary Nadler contraction condition fails.

Further work may investigate analogous conditions in b-metric spaces, partial metric spaces, modular metric spaces, graph-endowed metric spaces and other generalized distance structures. Recent literature continues to study multivalued contractions and stability in such extensions of classical metric fixed point theory.

Appendix: Major Theorems Used

This appendix collects, for reference, the statements of the principal classical fixed point theorems that motivate and underlie the results of this paper.

Theorem A.1 (Banach Contraction Principle, 1922)

Let (X, d) be a complete metric space and let f : X → X satisfy

$$d(f(x), f(y)) \le k\, d(x, y), \qquad 0 \le k < 1$$

for all x, y ∈ X. Then f has a unique fixed point in X.

Theorem A.2 (Nadler's Fixed Point Theorem, 1969)

Let (X, d) be a complete metric space and let F : X → CB(X) satisfy

$$H(F(x), F(y)) \le k\, d(x, y), \qquad 0 \le k < 1$$

for all x, y ∈ X, where H is the Hausdorff metric induced by d. Then F has at least one fixed point in X.

Theorem A.3 (Kannan's Fixed Point Theorem, 1968)

Let (X, d) be a complete metric space and let f : X → X satisfy

$$d(f(x), f(y)) \le k\big[d(x, f(x)) + d(y, f(y))\big], \qquad 0 \le k < \tfrac{1}{2}$$

for all x, y ∈ X. Then f has a unique fixed point in X.

Theorem A.4 (Mizoguchi–Takahashi Fixed Point Theorem, 1989)

Let (X, d) be a complete metric space and let F : X → CB(X). Suppose there exists a function φ : [0, ∞) → [0, 1) satisfying

$$\limsup_{r \to t^+} \varphi(r) < 1 \quad \text{for every } t \ge 0$$

such that

$$H(F(x), F(y)) \le \varphi\big(d(x, y)\big)\, d(x, y)$$

for all x, y ∈ X. Then F has at least one fixed point in X.

Theorem 3.1 of the present paper reduces to Theorem A.2 when b = 0 (Corollary 4.1), and its single-valued version (Corollary 4.3) recovers the essential conclusion of Theorem A.1. The Kannan-type consequence (Corollary 4.2) parallels the structure of Theorem A.3, while Theorem A.4 represents a different, function-controlled generalization of Theorem A.2 that is not directly subsumed by the hybrid condition (2.1).

References:

[1] Banach, S. (1922). Sur les opérations dans les ensembles abstraits et leur application aux équations intégrales. Fundamenta Mathematicae, 3, 133–181. https://doi.org/10.4064/fm-3-1-133-181.

[2] Bhimanatham, A., & Iyer, S. S. (2025). Evaluating the Impact of Recruitment Metrics on Employee Turnover and Retention Rates. International Journal of Research -Granthaalayah, 13(3), 387–392. https://doi.org/10.29121/granthaalayah.v13.i3.2025.6063

[3] Kannan, R. (1968). Some results on fixed points. Bulletin of the Calcutta Mathematical Society, 60, 71–76.

[4] Nadler, S. B., Jr. (1969). Multi-valued contraction mappings. Pacific Journal of Mathematics, 30(2), 475–488. https://doi.org/10.2140/pjm.1969.30.475.

[5] Covitz, H., & Nadler, S. B., Jr. (1970). Multi-valued contraction mappings in generalized metric spaces. Israel Journal of Mathematics, 8, 5–11. https://doi.org/10.1007/BF02771543.

[6] Paul, R., Yadav, D., Kishore, P., Kumar Sharma, I., Gulia, V., & Saini, P. (2025). A Cephalometric Study to Evaluate and Compare the Effect of Upper Premolar Extraction on Pharyngeal Airway Dimension in Skeletal Class I and Class Ii Patients. International Journal of Research -Granthaalayah, 13(5), 22–31. https://doi.org/10.29121/granthaalayah.v13.i5.2025.6169

[7] Reich, S. (1972). Fixed points of contractive functions. Bollettino dell'Unione Matematica Italiana, 5, 26–42.

[8] Mizoguchi, N., & Takahashi, W. (1989). Fixed point theorems for multivalued mappings on complete metric spaces. Journal of Mathematical Analysis and Applications, 141, 177–188. https://doi.org/10.1016/0022-247X(89)90214-X.

[9] Feng, Y., & Liu, S. (2006). Fixed point theorems for multi-valued contractive mappings and multi-valued Caristi type mappings. Journal of Mathematical Analysis and Applications, 317, 103–112. https://doi.org/10.1016/j.jmaa.2005.12.004.

[10] Klim, D., & Wardowski, D. (2007). Fixed point theorems for set-valued contractions in complete metric spaces. Journal of Mathematical Analysis and Applications, 334(1), 132–139. https://doi.org/10.1016/j.jmaa.2006.12.012.

[11] Mukherjee, G., Roy, M., & Dutta, S. (2025). Impact of Urbanization in Ekamra Kshetra, Odisha: Parametric Analysis of Variations. International Journal of Research -Granthaalayah, 13(7), 55–84. https://doi.org/10.29121/granthaalayah.v13.i7.2025.6276

[12] Abbas, M., Anjum, R., & Tahir, M. H. (2023). Fixed point theorems of enriched multivalued mappings via sequentially equivalent Hausdorff metric. Topological Algebra and its Applications, 11(1), 20220136. https://doi.org/10.1515/taa-2022-0136.

[13] Moga, M., & Truşcă, R. (2025). Fixed point and stability results for multi-valued nonlinear graph contractions in complete metric spaces. The Journal of Analysis, 33, 717–742.