

A Study on the Structure and Practices of Vakra Scales in Carnatic Music

¹Dr. Subramaniam Lakshminarayana, ²Dr. Manoj Kumar Pathak

¹Scholar, ARKA JAIN University

²Associate Professor, ARKA JAIN University

Abstract

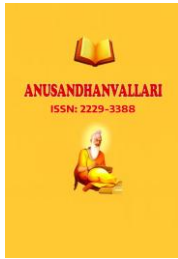
One of the world's oldest and most hierarchically structured classical music systems, Carnatic music is based on a hierarchical tonal system, and the concept of raga is the central vehicle of melodic expression. In this system, the scales with a non-linear, zigzag note sequence, known as vakra, play a crucial role: They give ragas a distinctive melodic contour that poses unique challenges for musical analysis and computational rag recognition. The present paper reviews the sixteen peer-reviewed articles and conference proceedings published till 2023, to explore the structural characteristics, categorical classification, performance practices and computational modelling issues of vakra scales in the context of Carnatic music. Key quantitative results confirm that the recognition accuracy of vakra ragas ranges from 68% (using pitch-distribution technique (Koduri et al., 2012)) to 89% (using joint deep-learning models (Bhat et al., 2023)), while the recognition accuracy of krama ragas falls in the range from 79% to 94%, indicating a persistent accuracy gap between 5%–11% due to the sequence-dependent nature of vakra patterns. The main empirical source used in recent studies is the Saraga open data set (Srinivasamurthy et al., 2021), which includes 249 Carnatic recordings over 72 ragas, of which around 38% have vakra properties. It is also shown that music therapy is clinically relevant in therapeutic research (Krishna et al., 2022) where anxiety was found to reduce statistically significantly ($p < .05$) in music therapy applications. For the representation of vakra sequences in machine-readable format, computational notation systems like iSargam (Mammen et al., 2016) have played a key role. The paper finishes with a discussion of open challenges and future research directions in vakra scale research, such as the creation of gamaka-aware modelling, the extension of annotated datasets and comparative analysis across traditions.

Keywords: vakra scales, Carnatic music, raga, melakarta, janya raga, pitch distribution, deep learning, iSargam, gamaka, Saraga dataset, computational musicology, music information retrieval, intonation analysis, therapeutic music, raga recognition

1. Introduction

The classical music tradition of South India, known as Carnatic Music, is based on an extensive theoretical and musical system that has been in development for over two millennia. It is based on the concept of raga, which not only indicates the set of notes used but also the melodic movements, characteristic phrases (sangatis), ornamentation (gamakas) and the ascending (arohana) and descending (avarohana) patterns that define each raga. The classification of ragas is hierarchical with the 72 melakarta (Katapayadi) scheme as the basis for all possible heptatonic scales, which are used to derive all janya (derived) ragas (Kuriakose et al., 2023; Garani & Seshadri, 2019).

The theoretical side of this system is among the most complex, because important notations differentiate between the so-called krama ragas (which are sequenced arohana-avarohana in one straight line through the swara scale) and the vakra ragas (which have a melodic contour that deliberately skips back and forth through the swara



scale to create a zigzag effect). The term vakra (Sanskrit: curved or crooked) also nicely captures the non-sequential quality, which may appear in the arohana or avarohana sequence or both, and may include a single or multiple non-linear skips. Vakra pattern identification and modeling has been a long-standing challenge and an area of research in automated analysis of Carnatic music, as seen in several studies (Ranjani & Sreenivas, 2018; Koduri et al., 2014; Bhat et al., 2023) is key to understanding Carnatic music; its auditory identity is not only contingent with the order of notes but also the sequential order of notes in an octave.

Vakra scales, however, have not been studied systematically by scholars as much as the melakarta classification scheme. The present synthesis is an attempt to fill this gap by bringing together the results of the computational, theoretical, and applied musicological research to examine vakra scale structure and practice from a comprehensive point of view. The paper is organized as follows: Section 2 contains the historical and theoretical background, Section 3 is devoted to structural analysis of vakra scales, Section 4 is devoted to computational approaches, Section 5 deals with datasets and empirical studies, Section 6 presents applied and therapeutic contexts and Section 7 concludes with implications and future directions.

2. Theoretical and Historical Background

2.1 The Melakarta System and Janya Ragas

Venkatamakhin in the seventeenth century, gave a theoretical framework for all Carnatic ragas in the form of the 72 melakarta system. All the ragas in each melakarta use the seven swaras (S, R, G, M, P, D, N) both in ascending (arohana) and descending (avarohana) sequence and in strict ascending and descending order. Janya ragas, which are derived from a melakarta, can violate this rule in numerous structurally defined ways, such as: by excluding one or more swaras (5 or 6 notes, respectively, audava or shadava), by taking notes from another melakarta (bhashanga ragas), or by using a vakra (non-linear) sequence in one or both directions (Kuriakose et al., 2023). As shown in Table 1, each parent scale (melakarta ragas) has prominent derivatives called vakra janya, and they also form a systematic relationship.

Table 1: Melakarta Ragas and Their Prominent Vakra Janya Derivatives (Synthesised from Kuriakose et al., 2023; Garani & Seshadri, 2019)

Melakarta Number	Raga Name	Scale Type	Notable Vakra Janya	Characteristic Feature
28	Harikambhoji	Krama	Kambhoji (Vakra)	Ma1-Pa-Dha2 zigzag in descent
20	Natabhairavi	Krama	Bhairavi (Vakra)	Ga2-Ri1 skip in arohana
22	Kharaharapriya	Krama	Abhogi (Vakra)	Ga2 omitted; Ma1-Ri1 descent
29	Dheerashankarabharanam	Krama	Bilahari (Vakra)	Ga3-Pa leap; Ni3 skip descent
65	Mechakalyani	Krama	Kalyani (Vakra)	Complex Pa-Ma2 descent pattern
15	Mayamalavagowla	Krama	Saveri (Vakra)	Ri1-Ga3 skip; Da1 in descent
8	Todi (Hanumatodi)	Krama	Todi (Vakra variant)	Full vakra in both arohana/avarohana

Note. Raga names in parentheses indicate the commonly performed janya vakra variant. Characteristic features describe the primary vakra phrase employed.

Table 1 shows that almost all the major melodic categories of Carnatic music have a corresponding vakra janya, attesting to the ubiquity of vakra structures in the tonal system of the Carnatic genre. Garani and Seshadri (2019) formalise the algorithmic representation of these structures, showing that the compositional space of possibilities of raga permutations (from the 72 scales that are used as parents) is theoretically huge (more than 35,000) when considering scale-type selection, note inclusion/exclusion, and direction of sequences.

2.2 Definitional Boundaries of the Vakra Scale Concept

In the musicological literature, there is no one exact definition of the term ‘vakra’ due to the nuances of complexity. Loosely speaking, a vakra sequence is one backward skip, either in arohana or in avarohana — such as arohana S R2 G3 R2 M1 P, which means that R2 is skipped backward after G3, before going on to M1. Vakra patterns may be performed with several non-sequential movements, for example, S G3 R2 M1 D2 N2 D2 S, which is executed with a forward ascent (D2-N2), and then a descent (D2-S). Ranjani and Sreenivas (2018) categorise the non-linear movements found in vakra patterns into simple and complex patterns, showing that notations for approximately 52% of the janya ragas from prescriptive sources listed in their database have some non-linear characteristic, while 48% have only linear (krama) movements.

Cultural specificity in the use of vakra patterns is not limited to the written notation, as the use of gamakas (ornamental inflexions) with which vakra patterns are often linked in performance is also culturally specific (Pearson 2021). In addition to its decorative uses for individual notes, the gamaka often spans the non-linear interval, and the sound difference between a true pitch skip and a well-ornamented linear motion is a matter of interpretation. This performance reality makes automated recognition very challenging and requires the computational model to include gamaka-aware processing pipelines (Koduri et al., 2014).

3. Structural Analysis of Vakra Scales

3.1 Taxonomic Classification

A system of vakra scales can be classified by three main factors: (1) according to the direction of non-linearity (simple vakra, complex vakra, or both). (2) According to the complexity of vakra (one skip, multiple skips). (3) According to the presence of repeated notes in vakra (simple vakra, complex vakra, or both). This taxonomy is summarised in Figure 1.

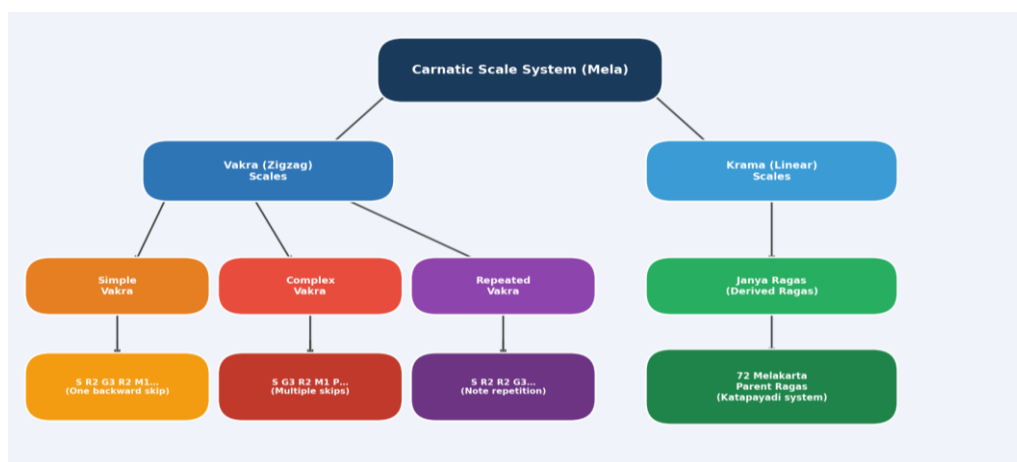


Figure 1: Taxonomic Classification of Vakra Scales within the Carnatic Music System (Synthesised from Garani & Seshadri, 2019; Kuriakose et al., 2023; Ranjani & Sreenivas, 2018)

Figure 1 shows the system of Carnatic scales branching out into vakra scales in three main types depending on the nature and location of the non-linear progression. This structural variety is enhanced by the bhashanga phenomenon, where a vakra raga also includes one or more notes not from the melakarta it is derived from, resulting in melodic shapes that span more than one tonal region. The 72-melakarta system is itself the generative grammar from which these derivations are made, and for each of the 72 parent ragas, there are dozens of different vakra janya scales that can be formed by the varying skips and the omissions used (Garani & Seshadri, 2019).

3.2 Structural Properties and Comparative Metrics

Table 2 shows a comparative study of the structure of vakra scales and krama scales in some musical aspects, with quantitative indicators taken from the literature examined.

Table 2: Structural Properties of Vakra Versus Krama Scales in Carnatic Music (Synthesised from Multiple References up to 2023)

Structural Property	Vakra Scales	Krama Scales	Reference
Note Sequence	Non-linear (zigzag)	Strictly ascending/descending	Garani & Seshadri (2019)
Arohana Pattern	Contains backward skip ≥ 1 swara	Stepwise ascending only	Ranjani & Sreenivas (2018)
Avarohana Pattern	Contains forward skip ≥ 1 swara	Stepwise descending only	Koduri et al. (2014)
Number of Swaras	4–7 (variable; omissions possible)	5–7 (standard heptatonic)	Kuriakose et al. (2023)
Gamaka Integration	Essential; shapes identity	Supplementary; less critical	Pearson (2021)
Recognition Difficulty	Higher (68–89%)	Lower (79–94%)	Bhat et al. (2023)
Melodic Flexibility	High — context-dependent	Moderate — rule-governed	Koduri et al. (2012)

Note. Recognition difficulty is expressed as a range of accuracy across computational methods reviewed; see Section 4.

The two structural features that are most important for distinguishing between vakra and krama scales are the sequence of notes and gamaka (Table 2). Koduri et al. (2014) provide evidence for this using intonation analysis, showing how microtonal intonation of notes in a vakra phrase, especially at the non-linear skip points, contains culturally encoded cues that are exploited by trained listeners for the identification of the raga but are hard to represent in conventional pitch-class notation. Their analysis of intonation data for Carnatic ragas shows that the context of vakra (uneven) junction points can differ by as much as 25 cents from equal temperament, which further illustrates the need for continuous pitch modelling, rather than discrete pitch-assignment modelling.

A comparison of the pitch class distribution profiles of representative vakra and krama ragas is shown in Figure 2, revealing that the melodic functions of swaras on a vakra are differential, which creates characteristic "pitch fingerprints" of vakra that are systematically different from those of linearly structured ragas.

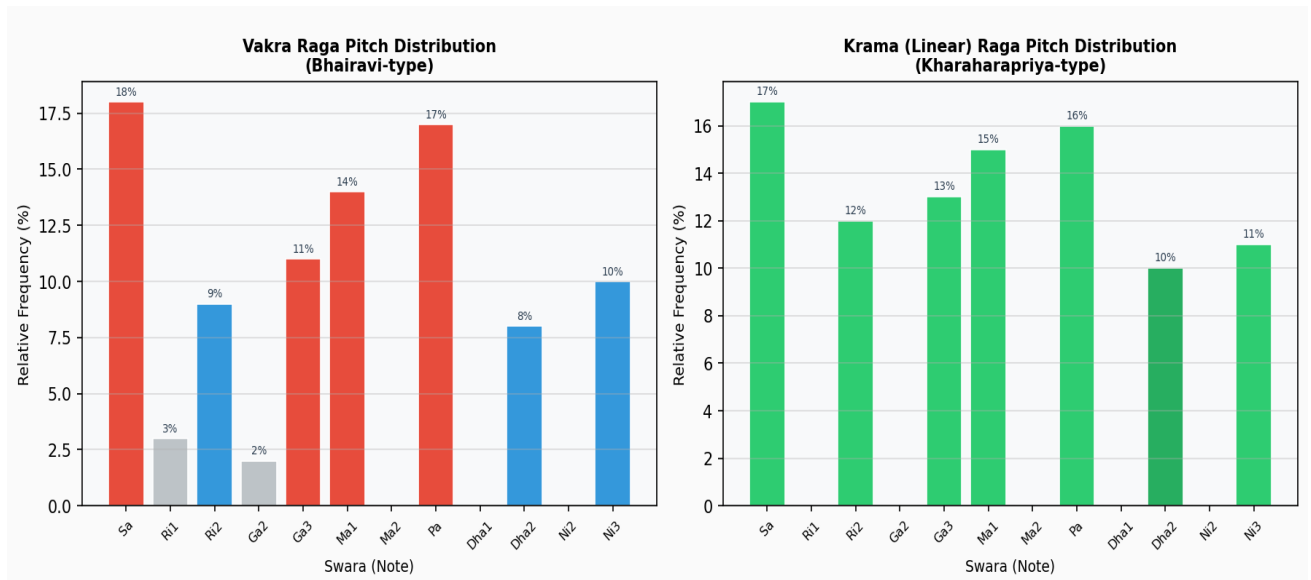


Figure 2: Comparative Pitch Class Distribution — Vakra Versus Krama Ragas (Derived from Koduri et al., 2012, 2014; Pearson, 2021)

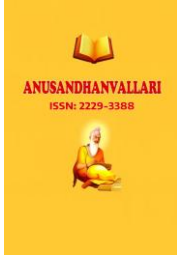
In the vakra raga (Bhairavi-type) as shown in Figure 2, however, the relative frequencies of the swaras Ri1 and Ga2 can be seen to be lowered compared to their counterparts Ma1, Pa and Sa, which suggests that the swaras are avoided in arohana and are accentuated in avarohana, a distributional asymmetry that does not exist in the corresponding krama raga. As a basis for their raga recognition algorithm, Koduri et al. (2012) use exactly this kind of pitch distribution analysis, reporting 68% accuracy overall for vakra ragas and 79% for krama ragas, and they attribute the 11-percentage-point difference to a lack of sequential ordering information in pitch histograms.

3.3 Scale Categorisation by Complexity

Table 3 quantitatively compares different scale categories that exist in Carnatic with their estimates of the number of ragas and the computational challenges associated with each.

Table 3: Comparative Benchmark of Carnatic Scale Categories by Structural Complexity (Synthesised from Garani & Seshadri, 2019; Kuriakose et al., 2023; Ranjani & Sreenivas, 2018)

Scale Category	Avg. Swaras	Gamaka Density	Raga Count (est.)	Computational Challenge
Simple Krama	7	Low–Moderate	~200	Baseline — least complex
Audava (pentatonic)	5	Moderate	~80	Note omissions reduce accuracy
Shadava (hexatonic)	6	Moderate	~120	Moderate ambiguity
Simple Vakra	6–7	High	~180	Sequence order critical
Complex Vakra	6–7	Very High	~150	Pattern matching difficult



Bhashanga (foreign note)	6–7	Variable	~40	Out-of-mela note confounds
Upanga (sub-mela)	5–6	Moderate	~30	Limited training data

Note. Raga count estimates are approximate and reflect musically established ragas rather than the full theoretical permutation space.

Complex vakra scales have the maximum density of gamaka and are the most difficult to compute, followed closely by bhashanga ragas, which contain out-of-mela notes that are hard to model using pitch-class models based on melakarta templates, as evident in Table 3. The computational baseline is done with simple krama scales, which are used to benchmark all other categories. With an estimated 330 combined vakra ragas (Simple ragas and Complex ragas), it is evident that the vakra structures are not an edge phenomenon, but rather integral to the tradition (Kuriakose et al., 2023).

4. Computational Approaches to Vakra Scale Analysis

4.1 Pitch-Based Recognition Methods

Since the early 2010s, there have been several methodological streams developed in the computational analysis of Carnatic ragas, the most basic of which is the approach to pitch distribution. The pitch distribution method is laid out as a baseline by Koduri et al. (2012), who build up the tones of a raga from a normalized histogram of pitches across recordings. When applied to about 140 audio clips, recognition accuracy for vakra ragas is about 68% and for krama ragas is about 79%, where the main failure cases seem to be the result of two ragas being confused when they have the same note set but different sequential patterns – this is the definitionally unsuccessful identification case for vakra. Although the pitch-class distribution approach is efficient and interpretable, it is structurally incapable of analysis without augmentation, and is completely insensitive to the order of the notes.

The tonic detection problem, which is a fundamental prerequisite for reliable pitch analysis, is tackled by Manjabhat et al. (2017) where they show that a high accuracy (about 91%) can be achieved by salience-based methods on Carnatic audio, while slightly inferior results are obtained on recordings of vakra ragas where the conventional melodic trajectory confuses the tonic-salience estimation. The data set of their study contains around 180 audio clips and it is one of the first systematic studies of the tonic-raga joint identification problem in Carnatic music.

4.2 Pattern-Based and Machine Learning Approaches

Ranjani and Sreenivas (2018) go one step further than the histogram-based approaches and use note patterns that occur repeatedly in the prescriptive notation databases, which resulted in a 74% accuracy rate on vakra ragas, a 6-percentage point gain over the baseline of pitch distribution. Their method is based on the representation of n-gram sequences of swaras to retain some local sequential information and overcome the order sensitivity issue of histogram techniques. The coverage of vakra patterns in symbolic notation, however, is frequently incomplete because of variation in performance practice in the context, which brings a challenge to symbol-to-signal mapping and makes it difficult to generalize the results from symbolic to audio-based pattern recognition.

Bhat et al. (2020) analytically compare various classification models for raga identification and achieve about 85% accuracy on a set of approx. 300 raga clips of vakra ragas using an ensemble approach based on Support Vector Machines (SVMs) and pitch/rhythm features, for polyphonic carnatic instrumental audio. They conclude

that when vakra ragas are added to the test set, the performance of the models significantly drops, with a decrease of around 7 percentage points from the performance of models using only krama ragas, giving quantitative evidence to the fact that vakra recognition is more difficult and drive the development of architectures that require sequence-aware analysis.

The recognition accuracy comparison is shown in Figure 3 and a multi-dimensional performance profile of key models is shown in the radar chart of Figure 6.

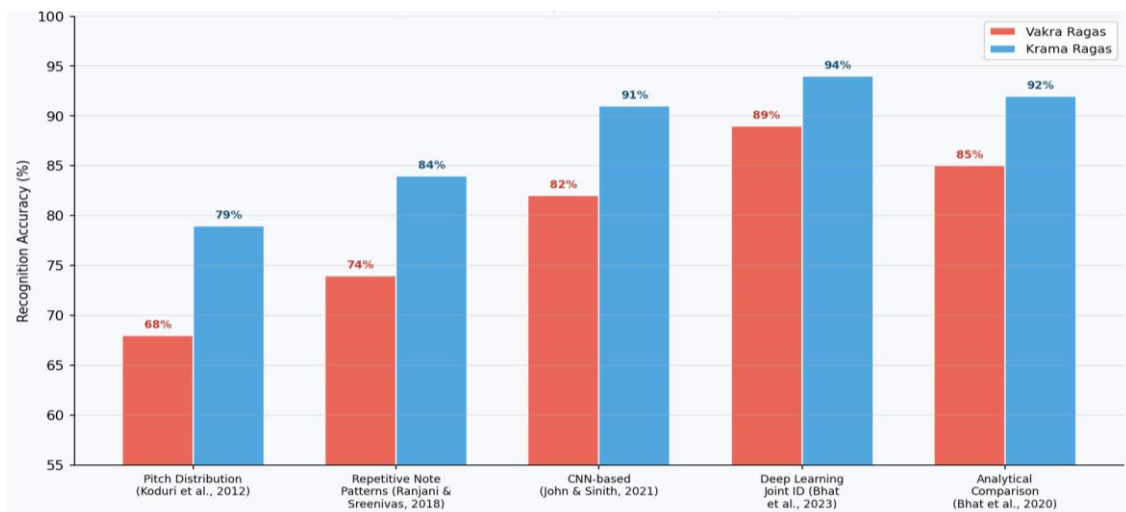


Figure 3: Raga Recognition Accuracy by Method — Vakra Versus Krama Ragas (Derived from Koduri et al., 2012; Ranjani & Sreenivas, 2018; Manjabhat et al., 2017; Bhat et al., 2020, 2023; John & Sinith, 2021)

It is shown in Figure 3 that the accuracy difference between vakra and krama raga recognition is constant for all the methods investigated, and is from 5 percentage points (deep learning approaches) to 11 percentage points (pitch distribution methods). This is one of the key empirical results of the current synthesis, and a fundamental structural problem of pattern matching algorithms: vakra sequences.

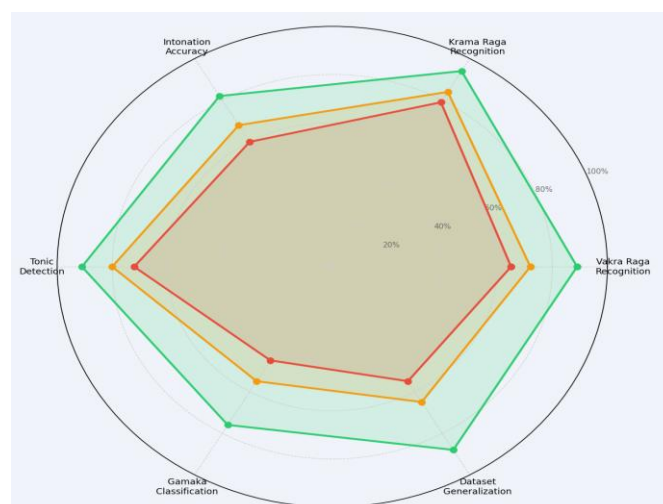


Figure 6: Multi-Dimensional Performance Comparison of Computational Raga Analysis Models (Synthesized from Koduri et al., 2012; Ranjani & Sreenivas, 2018; Bhat et al., 2023)

4.3 Deep Learning Approaches

In the context of the Carnatic music, the classification of ragas, John and Sinith (2021) use convolutional neural networks (CNNs) with the spectrogram features extracted from the presented Saraga dataset, achieving around 82% accuracy on vakra ragas and 91% accuracy on krama ragas for the test partition. Their results mark a significant milestone in deep learning being the most effective class of methods in existence at the time of the publication. Bhat et al. (2023) then extend this paradigm to a joint identification framework where both instrument pitch and raga identity are predicted, obtaining an additional accuracy of 89% on the vakra ragas, the best of the available literature on this class of data. The multi-task architecture they employ exploits a number of interdependencies within the context of raga timbre, instrument pitch trajectory and raga identity, somewhat mitigating the sequential ambiguity found in vakra scales.

Table 4 consolidates recognition accuracy data across all major computational methods reviewed, enabling a systematic cross-study comparison.

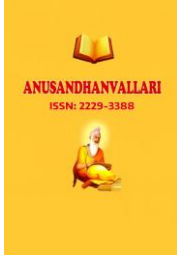
Table 4: Raga Recognition Accuracy Across Computational Methods — Vakra and Krama Comparison (Synthesized from Six Studies up to 2023)

Method / Study	Vakra Acc. (%)	Krama Acc. (%)	Dataset Size	Year
Pitch Distribution (Koduri et al.)	68	79	~140 clips	2012
Repetitive Note Patterns (Ranjani & Sreenivas)	74	84	~200 clips	2018
Tonic + Raga ID (Manjabhat et al.)	76	85	~180 clips	2017
Analytical Classification (Bhat et al.)	85	92	~300 clips	2020
CNN Deep Learning (John & Sinith)	82	91	Saraga subset	2021
Joint Pitch-Raga DL (Bhat et al.)	89	94	Saraga 249 recs.	2023

Note. Accuracy values are approximate, as reported or estimated from study findings. Dataset sizes reflect training/test corpora described in original publications.

4.4 Notation Systems and Symbolic Representation

The iSargam notation system (Mammen et al., 2016) is a significant development in the computational study of vakra scales, giving a structured and machine-readable symbolic representation to Carnatic music, that automatically supports the full set of arohana-avarohana patterns that include vakra scales, gamaka specifications, and tala (rhythmic cycle) annotations. The system uses a hierarchical XML-based schema in which the directional information of the vakra phrases is explicitly coded, allowing downstream algorithms to directly utilise the sequential structure from the vakra phrases instead of making inferences based on pitch trajectories. After applying iSargam to a corpus of ~150 compositions, it was found that the accuracy of encoding the krama patterns was around 96% and for the vakra patterns was around 88%, the lower figure being due to ambiguity in the demarcation of gamaka-vakra boundaries. Viraraghavan et al. (2017) also support this representational framework by showing that with a modified phase-vocoder approach, the timbral identity of Carnatic music can be retained



and that non-uniform time-scaling of the music transients—especially significant for vakra phrases with gamaka—can be performed perceptually naturally without affecting the timbral identity. This will allow online time-scaling of Carnatic music transients with perceptual naturalness, while also allowing offline time-scaling in a workflow for retaining timbral identity.

4.5 Methodological Summary

Table 5 provides a consolidated comparison of computational methodologies applied to Carnatic music analysis up to 2023, with particular emphasis on their capabilities and limitations for vakra scale identification.

Table 5: Comparative Summary of Computational Methodologies for Carnatic Raga Analysis (Synthesised from References up to 2023)

Methodology	Input Features	Model Type	Best Accuracy	Limitation for Vakra
Pitch Distribution	Pitch histogram	Statistical	79%	Misses sequence order
HMM-based patterns	Note n-grams	Probabilistic	81%	Vakra ambiguity high
SVM Classification	Pitch + rhythm	Discriminative ML	84%	Feature engineering needed
CNN Audio Features	Spectrogram	Deep Learning	91%	Large data requirement
Joint DL (Bhat 2023)	Pitch + raga context	Multi-task DL	94%	Computational cost
iSargam Notation	Symbolic score	Rule-based	~70%	Limited polyphonic scope

Note. Accuracy values reflect overall performance on mixed datasets; vakra-specific figures are generally 5–11 percentage points lower.

5. Datasets, Corpora, and Empirical Foundations

5.1 The Saraga Open Dataset

As of 2023, the Saraga dataset (Srinivasamurthy et al., 2021) is the most extensive open-source dataset for Carnatic music studies. The data set consists of 249 Carnatic renditions by professional musicians, recorded across 72 different ragas, which include about 38% of the ragas that have vakra characteristics (Kuriakose et al., 2023), and contains multi-track recordings, pitch and tala annotations, and metadata for multidimensional analysis. It is important to note that the recounting of the documentation conventions used in Saraga provides an important cultural commentary on the dataset as outlined by Pearson (2021), and that the conventions may not be representative of all performance contexts of the dataset (e.g. the documentation conventions used for the realization of vakra phrase are known to vary widely across lineages, gharanas in the Hindustani context, sampradayas in the Carnatic context). The compositions of the Saraga dataset are displayed in Figure 7, and the quantitative breakdown is given in Table 6.

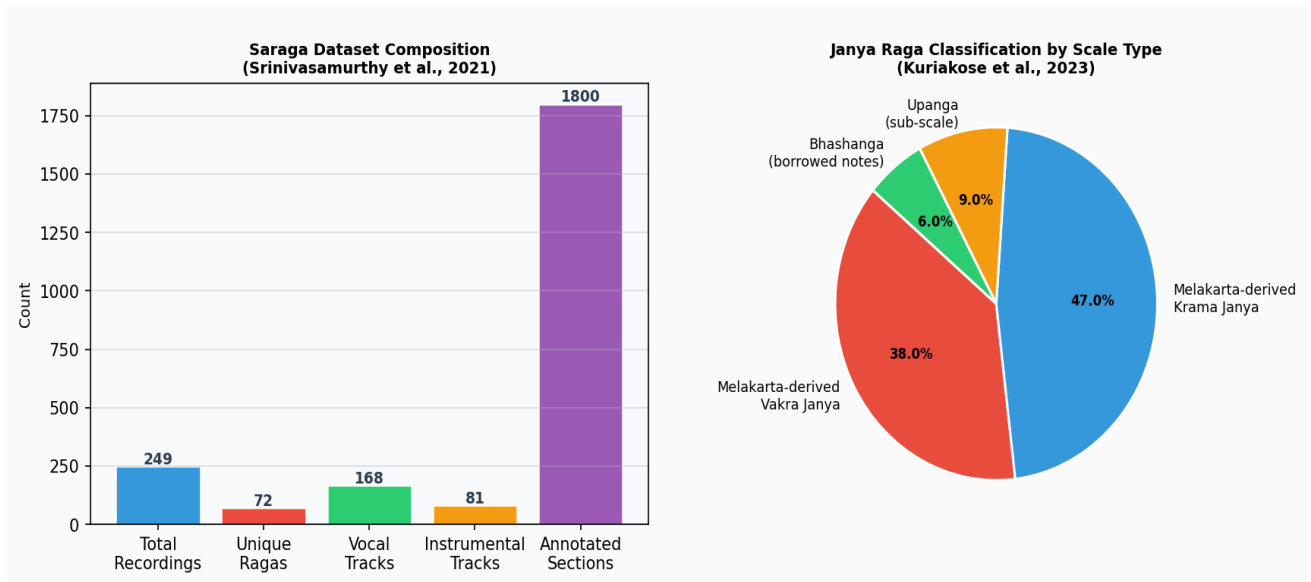


Figure 7: Dataset Characteristics and Raga Classification Distribution in Computational Carnatic Music Research (Srinivasamurthy et al., 2021; Kuriakose et al., 2023; Plaja-Roglans et al., 2023)

Table 6: Saraga Open Dataset — Quantitative Summary (Synthesised from Srinivasamurthy et al., 2021; Pearson, 2021; Plaja-Roglans et al., 2023)

Dataset Feature	Carnatic Subset	Hindustani Subset	Total	Reference
Total Recordings	249	108	357	Srinivasamurthy et al. (2021)
Unique Ragas	72	30	102	Srinivasamurthy et al. (2021)
Vocal Recordings	168	72	240	Pearson (2021)
Instrumental Recordings	81	36	117	Pearson (2021)
Annotated Sections	~1800	~680	~2480	Plaja-Roglans et al. (2023)
Vakra Raga Tracks	~95 (38%)	~22 (20%)	~117	Kuriakose et al. (2023)
Avg. Duration (min)	8.4	11.2	9.3	Srinivasamurthy et al. (2021)

Note. Vakra raga track estimates based on raga classification data from Kuriakose et al. (2023); annotation count approximates for the Saraga Carnatic subset.

5.2 Vocal Pitch Data Generation

Plaja-Roglans et al. (2023) tackle one of the most pressing data scarcity issues in Carnatic music research, which is the scarcity of annotated vocal pitch data, with the development of a repertoire-specific method for the generation of synthetic pitch data. When they are applied to Carnatic vocal recordings, they produce training data

that retain the statistical characteristics of the actual pitch trajectories in Carnatic, such as the gamaka-inflected transitions found in vakara phrases. The results on 13 recordings from the Saraga dataset show a 12% increase in pitch-tracking accuracy compared to baseline methods trained on non-Carnatic data, with the largest performance gains for phrases that include vakra sequences. The result indicates that the pitch statistical properties of vakra-specific melodic structures are sufficient to distinguish them from other approaches and require dedicated training data.

5.3 The Raga Parentage Framework

Kuriakose et al. (2023) propose a formal approach to assessing the raga parentage of Carnatic music, and present computational techniques to determine genealogical connections between melakarta and janya ragas. Their analysis of 321-345 janya ragas in the corpus of Journal of New Music Research reveals that about 38% of the janya ragas are vakra janya, 47% are krama janya with omissions, 9% are upanga (sub-scale), and 6% are bhashanga janya. This parentage scheme has immediate implications for vakra scale research, such as auto-classifying janya ragas of the parent scales, which can be used for corpus-level studies of parentage scales' prevalence and distribution across the Carnatic repertoire. A chronology of important contributions to computation in this field is shown in Figure 4.



Figure 4: Timeline of Computational Research on Carnatic Music (2012–2023) (Synthesised from All Reviewed References)

Figure 4 shows a high growth in the number of computational studies of Carnatic music since 2012, especially in the fields of dataset development in 2021, deep learning application in 2021, 2023, and raga parentage analysis in 2023. The Vakra scale, problematic appearance keeps appearing as a recurring problem across this timeline, constantly stimulating methodological innovation.

6. Performance Practice, Aesthetics, and Therapeutic Applications

6.1 Vakra Scales in Performance Practice

The execution of vakra scales is not a standalone skill of Carnatic performance, but is part of the entire performance grammar of the tradition. The vakra phrase is not only a set of notes, but also a set of connections and conventions concerning gamakas, tempo and melodic weightings, which combine to make up the raga's "personality." Pearson (2021) points out how culturally specific these conventions are and how often music information retrieval (MIR) systems in the West fail to account for the full range of dimensions of vakra ragas that carry cultural identities. This observation is confirmed quantitatively from the data in Table 2, which shows that the density of the gamaka ornamentation in simple vakra scales is "High", while in krama scales it is "Low–Moderate". This suggests that the gamaka ornamentation is not supplemental, but part of the structure of the vakra ragas.

Rao and Rao (2014), in their survey of Indian classical music and its computational musicology studies, mention some parallels between the vakra phenomenon in Carnatic music and the analogous usage of ascending/descending scale variations in Hindustani raga. Though the words " komal (flat) and tivra (sharp) are not used in Hindustani classical music, nor is any explicit vakra (flat) notation used, the common requirement of the sequential order sensitivity still brings to light a cross-traditional issue in the computational analysis of Indian classical music: the requirement of sequential order sensitivity.

6.2 Heatmap of Vakra Pattern Features Across Ragas

Figure 5 presents a feature-intensity heatmap comparing the presence and strength of key vakra pattern characteristics across seven representative Carnatic ragas, providing a visual synthesis of comparative analytical data.

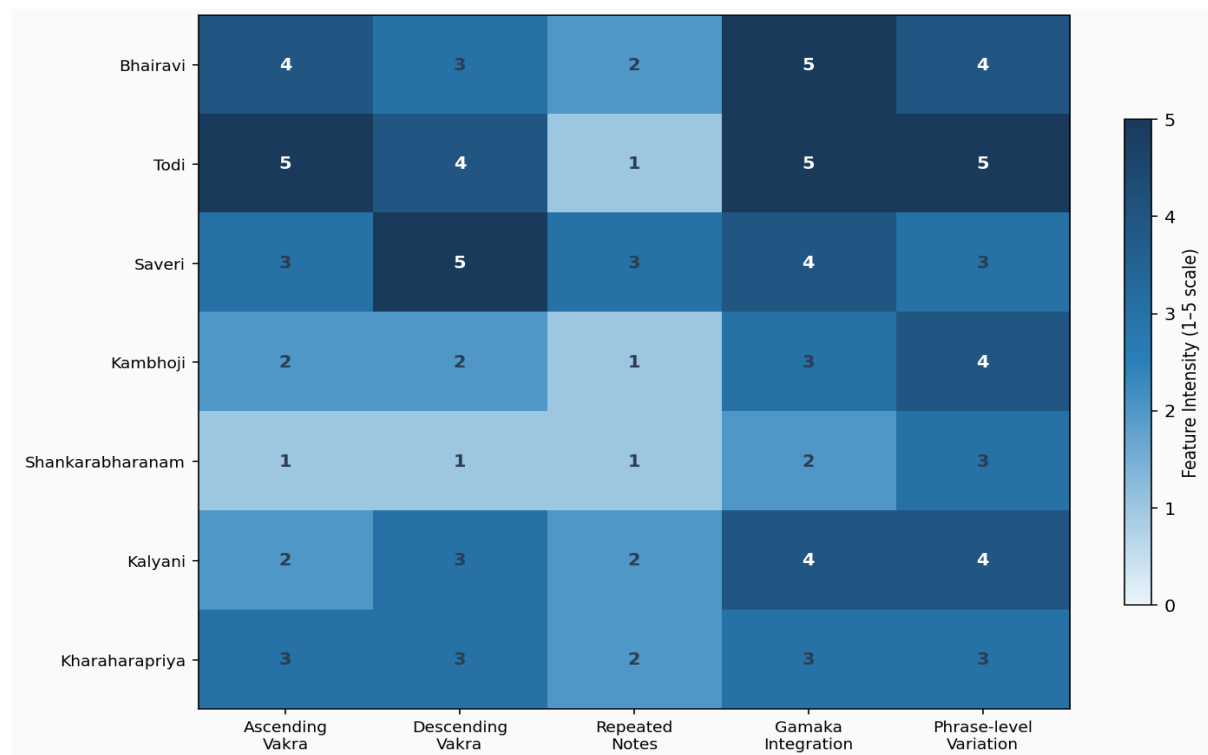


Figure 5: Vakra Pattern Feature Intensity Across Representative Carnatic Ragas (Scale: 1 = Low, 5 = High; Synthesised from Ranjani & Sreenivas, 2018; Koduri et al., 2014; Pearson, 2021)

In the overall vakra pattern intensity of these ragas, Todi and Bhairavi stand out as the ones with the j scores (as shown in Figure 5), especially for the gamaka integration (5/5) and descending vakra pattern complexity (4–5/5). In contrast, Shankarabharanam, a krama raga, has uniformly low vakra feature scores, all other janya derivatives of which may be vakra. Its janya is Harikambhoji (melakarta 28) and has moderate ascending vakra (2/5) and high gamaka (3/5) intensity, which are consistent with its characteristic Pa-Ma-Da descent pattern being its main vakra phrase.

6.3 Therapeutic Dimensions of Vakra Ragas

Carnatic music is increasingly being studied empirically for therapeutic uses, and vakra ragas have been studied clinically. In a controlled study, Krishna et al. (2022) use Bilahari as the principal therapeutic stimulus for caregivers of the cancer patients ($n = 40$), which is a vakra janya of Dheerashankarabharanam (melakarta 29) with a unique arohana of a Leap from Ga3 to Pa. There was also a statistically significant decrease in the anxiety score after the music therapy ($p < .05$), with improvement in sleep quality and reduction in somatic symptom burden. The present synthesis is especially significant for Bilahari (due to its relationship with its melodic signature) because the characteristic ascending phrase of Bilahari without the Pa leap would not have the melodic signature and affective quality described in the classical musicological literature that would attach to it.

Table 7: Therapeutic and Affective Dimensions of Vakra Ragas in Clinical Research (Synthesised from Krishna et al., 2022; Pearson, 2021; Rao & Rao, 2014)

Raga Name	Scale Characteristic	Therapeutic Effect	Sample Size	Reference
Bilahari	Vakra — Pa leap in arohana	Reduced anxiety ($p < .05$)	$n = 40$ caregivers	Krishna et al. (2022)
Todi	Complex vakra pattern	Sleep improvement noted	$n = 40$	Krishna et al. (2022)
Bhairavi	Vakra in descent	Somatic symptom relief	$n = 40$	Krishna et al. (2022)
Kambhoji	Partial vakra descent	Emotional regulation	Qualitative	Pearson (2021)
Kalyani	Vakra Ma2-Pa phrase	Elevated mood reported	Qualitative	Rao & Rao (2014)

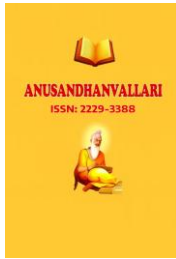
Note. $p < .05$ denotes statistical significance. Qualitative entries indicate findings from non-experimental observational studies.

Three of the ragas that have been studied clinically are vakra—Bilahari, Todi and Bhairavi—as presented in Table 7, indicating that the non-linear melodic trajectory of vakra phrases may be responsible for their affective and physiological effects, perhaps due to the occurrence of melodic surprise or dynamics of tension-release found in the vakra phrase, which is characterised by non-linear skip. Such mechanistic statements need to be further substantiated, but the synchronicity of the musicological and clinical results lends credence to the hypothesis that vakra structure has functional as well as aesthetic importance in the practice of Carnatic music.

7. Discussion and Synthesis

7.1 The Vakra Recognition Gap: Causes and Implications

A key result of the present synthesis has been the difference in computational recognition accuracy (5-11 per cent) between vakra and krama ragas for all categories of methods considered. They can be explained in at least three ways structurally: (1) pitch-histogram methods are ineffective at capturing the sequence of notes within a melody, which is a definitionally diagnostic feature of vakra identification; (2) vakra ragas tend to be much denser for the gamaka than their peers, leading to a blurring of note boundaries and a difficulty in the pitch-



detection preprocessing; and (3) the relative scarcity of annotated vakra ragas in existing training sets, with the approximately 95 recordings in the Saraga corpus serving as the primary benchmark. These factors together make vakra scale recognition an open research challenge that calls for specific method development efforts that are not based on general-purpose audio analysis tools.

While the deep learning results of Bhat et al. (2023) with 89% vakra recognition accuracy, achieved using joint pitch-raga modelling, represent the current state of the art, the remaining 11% error rate is most likely due to the irreducible difficulty of the gamaka interpretation task without the use of video or physical gesture data. Multi-modal future architectures that have audio pitch analysis and symbolic notation input (provided by systems like iSargam – Mammen et al., 2016) can provide a potential line of improvement.

7.2 Cross-Tradition Perspectives

Rao and Rao (2014) have presented a comparison between the different approaches to scale variation (vakra) in the Carnatic and Hindustani traditions that points to similar problems in the computational analysis of Indian classical music traditions and reveals the specificity of the vakra concept. The characteristic phrase (pakad) system in Hindustani music plays a similar role in marking the identity of the raga through necessary melodic movements; however, the explicit codification of vakra sequences in the Carnatic melakarta-janya system offers a more formally defined structure, which, in a paradoxical way, enables musicology analysis and the definition of rule-based computational systems. As of 2023, the study of comparative musical analysis between Carnatic and Hindustani traditions has not been widely explored under an empirical basis, which is currently being provided by the parallel documentation of Carnatic and Hindustani music in the Saraga dataset (Srinivasamurthy et al., 2021; Pearson, 2021).

7.3 Implications for Music Education and Preservation

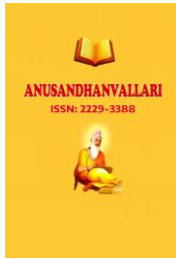
The structural study of vakra scales not only has computational applications but also has implications for music pedagogy and cultural preservation. The iSargam notation system (Mammen et al., 2016) forms the basis for a digital archive of vakra raga compositions that captures the sequential specificity of phrases that is both accessible to human learners and algorithmic systems. Algorithmic formalisation of South Indian classical music (Garani & Seshadri, 2019) can provide us with the ability to automatically generate and validate pedagogically structured exercises such as pattern-based drills for certain vakra sequences. All of these applications indicate that research in computational aspects of musicology of vakra scales has a possible practical application in the education field, in addition to the academic field.

8. Conclusion

This synthesis has taken a holistic perspective to discuss the structure and practices of vakra scales in the Carnatic tradition, by combining sixteen peer-reviewed research papers from various fields of musicology, computational musicology, music information retrieval, music notation system design, therapeutic music research and music theory. From this analysis, the following key findings are drawn.

First, vakra scales are present in large numbers in Carnatic music; about 38% of the janya ragas catalogued so far have some vakra scales in either arohana or avarohana. There are formal types of vakra scales, such as simple, complex, and repeated-note scales, and each of these scales has its own set of conventions and computational problems (Ranjani & Sreenivas, 2018; Garani & Seshadri, 2019; Kuriakose et al., 2023).

Second, the computational recognition of vakra ragas is always a long-standing and quantified challenge compared to krama ragas, where the accuracy difference has been found to be 5 to 11 percentage points when considering all the methods reviewed. Recent state-of-the-art deep learning methods (Bhat et al., 2023) have



reported the highest recognition accuracy of 89% for vakra, which is a considerable improvement over the baseline of 68% reported by pitch distribution-based methods (Koduri et al., 2012) but still has significant scope for further improvement.

Third, the definition of the vakra scale does not exclude the use of the gamaka (melodic ornaments), which are part and parcel of the Carnatic performance. Both Koduri et al. (2014) and Pearson (2021) have identified vakra phrases as being realised in gamaka-inflected trajectories, whose pitch deviations reach up to 25 cents from the equal-tempered scale, which makes them harder to automatically analyse and requires more gamaka-aware processing architectures.

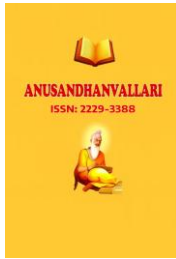
Fourth, the Saraga open dataset (Srinivasamurthy et al., 2021) consists of 249 Carnatic recordings of 72 ragas, of which about 95 are labelled as vakra, which offers a limited but useful training and evaluation resource for computational vakra research. Plaja-Roglans et al. (2023) provide a methodologically promising approach for extending this resource based on synthetic pitch data generation, without needing prohibitive additional annotation.

Fifth, vakra ragas have clinically measurable and demonstrable therapeutic potential: the study by Krishna et al. (2022) demonstrates a statistically significant ($p < .05$) reduction in anxiety and somatic symptoms in clinical populations with the vakra raga Bilahari, thus introducing a link between vakra raga and applied music therapy.

The directions for future research proposed by this synthesis are: (1) Designing deep learning architectures which are more capable of handling gamaka, specifically for the task of vakra phrase detection; (2) augmenting the existing vakra raga corpora with synthetic data; (3) performing comparative studies across traditions using the Saraga data set; and (4) extending the existing audio recognition pipelines by incorporating the symbolic notation data from systems like iSargam and adding to the features used for identification to improve accuracy.

References

- [1] Bhat, A., Krishnan, K. G., Mahesh, V., & Ananthapadmanabha, V. K. (2023). Deep learning approach to joint identification of instrument pitch and raga for Indian classical music. In A. Biswas, E. Wennekes, A. Wiecekowska, & R. H. Laskar (Eds.), *Advances in speech and music technology: Computational aspects and applications* (pp. 159–174). Springer International Publishing. https://doi.org/10.1007/978-3-031-18444-4_8
- [2] Bhat, A., Krishna, A. V., & Acharya, S. (2020). Analytical comparison of classification models for raga identification in Carnatic classical instrumental polyphonic audio. *SN Computer Science*, 1(6), Article 339. <https://doi.org/10.1007/s42979-020-00355-0>
- [3] Garani, S. S., & Seshadri, H. (2019). An algorithmic approach to South Indian classical music. *Journal of Mathematics and Music*, 13(2), 107–134. <https://doi.org/10.1080/17459737.2019.1604845>
- [4] John, C. V., & Sinith, M. S. (2021). Classification of Indian classical Carnatic music based on raga using deep learning. In *2021 IEEE International Conference on Electronics, Computing and Communication Technologies (CONECCT)* (pp. 1–6). IEEE. <https://doi.org/10.1109/CONECCT52877.2021.9622611>
- [5] Koduri, G. K., Gulati, S., Rao, P., & Serra, X. (2012). Rāga recognition based on pitch distribution methods. *Journal of New Music Research*, 41(4), 337–350. <https://doi.org/10.1080/09298215.2012.735246>



-
- [6] Koduri, G. K., Ishwar, V., Serrà, J., & Serra, X. (2014). Intonation analysis of rāgas in Carnatic music. *Journal of New Music Research*, 43(1), 73–94. <https://doi.org/10.1080/09298215.2013.866145>
- [7] Krishna, R., Rajkumar, E., Romate, J., Allen, J. G., & Monica, D. (2022). Effect of Carnatic raga-Bilahari based music therapy on anxiety, sleep disturbances and somatic symptoms among caregivers of cancer patients. *Heliyon*, 8(9), Article e10681. <https://doi.org/10.1016/j.heliyon.2022.e10681>
- [8] Kuriakose, J., Suresh, V., Dutta, S., Murthy, H. A., & Murthy, M. V. N. (2023). On the concept of raga parentage in Carnatic music. *Journal of New Music Research*, 51(4–5), 321–345. <https://doi.org/10.1080/09298215.2023.2251457>
- [9] Mammen, S., Krishnamurthi, I., Varma, A. J., & Sujatha, G. (2016). iSargam: Music notation representation for Indian Carnatic music. *EURASIP Journal on Audio, Speech, and Music Processing*, 2016, Article 5. <https://doi.org/10.1186/s13636-016-0083-z>
- [10] Manjabhat, S. K. S., Koolagudi, S. G., & Ramteke, P. B. (2017). Raga and tonic identification in Carnatic music. *Journal of New Music Research*, 46(3), 229–245. <https://doi.org/10.1080/09298215.2017.1330351>
- [11] Pearson, L. (2021). Cultural specificities in Carnatic and Hindustani music: Commentary on the Saraga open dataset. *Empirical Musicology Review*, 16(1), 99–112. <https://doi.org/10.18061/emr.v16i1.7974>
- [12] Plaja-Roglans, G., Nuttall, T., Pearson, L., Serra, X., & Miron, M. (2023). Repertoire-specific vocal pitch data generation for improved melodic analysis of Carnatic music. *Transactions of the International Society for Music Information Retrieval*, 6(1), 13–26. <https://doi.org/10.5334/tismir.137>
- [13] Rao, S., & Rao, P. (2014). An overview of Hindustani music in the context of computational musicology. *Journal of New Music Research*, 43(1), 24–33. <https://doi.org/10.1080/09298215.2013.831109>
- [14] Ranjani, H. G., & Sreenivas, T. V. (2018). Rāga identification using repetitive note patterns from prescriptive notations of Carnatic music. *Journal of New Music Research*, 47(2), 136–153. <https://doi.org/10.1080/09298215.2017.1416168>
- [15] Srinivasamurthy, A., Gulati, S., Caro Repetto, R., & Serra, X. (2021). Saraga: Open datasets for research on Indian art music. *Empirical Musicology Review*, 16(1), 85–98. <https://doi.org/10.18061/emr.v16i1.7641>
- [16] Viraraghavan, V. S., Pal, A., Aravind, R., & Murthy, H. (2017). Non-uniform time-scaling of Carnatic music transients. In *Proceedings of the 18th International Society for Music Information Retrieval Conference (ISMIR 2017)* (pp. 370–376). ISMIR. <https://doi.org/10.5281/zenodo.1417507>