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Divided Square Difference Cordial Labeling of Some Cycle Related Graphs

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Abstract

In this paper we prove the following results:

1. The path union of two copies of cycle Cn (n∈N, n3) is a divided square difference cordial graph.
2. The path union of two copies of cycle C1,1,n-2 (n N, n5) with three chords is a divided square difference cordial graph, where three chords which by themselves forms a triangle.
3. The path union of two copies of wheel Wn(n∊N, n3) is a divided square difference cordial graph.
4. The path union of two copies of helm Hn (n∊N, n3) is a divided square difference cordial graph.
5. The path union of two copies of closed helm CHn(n∊N, n3) is a divided square difference cordial graph.

MSC 2010 Classifications: 05C78.

Keywords: Sum divisor cordial labeling, Sum divisor cordial graph, path union of two graphs.

Introduction

All the way through this paper, by a graph we meane a finite, undirected, simple graph G =(V, E) with p vertices and q edges. We follow the central annotations and phraseology of graph theory as in [6] and number theory as in [3]. Labeling of a graph is a mapping that connects the graph elements to the numbers set, generally to the set of natural numbers. If the domain is a collection of nodes the labeling is called node labeling. If the domain is the collection of edges, then we allude to edge labeling. If the labels are given to both nodes and edges then the labeling is called total labeling. Graph Labeling is an energetic region that belongs to research in graph theory. The concept of graph labeling was introduced by Rosa. J. A. Gallian [5] developed graph labeling which performs as a boundary stuck flanked between number theory and the structure of graphs.
The present work is aimed to discuss one such labeling known as divided square difference cordial labeling.

Main Results

Theorem 1. The path union of two copies of cycle C_n $(n \in N, n \geq 3)$ is a divided square difference cordial graph.
Proof. Let G = (2. C_n) be the path union of two copies of cycle C_n. Let $v_1, v_2, \dots, v_n$ be the successive vertices of C_n and let $u_1 = v_{n+1}, u_2 = v_{n+2}, \dots, u_n = v_{2n}$ be the successive vertices of another C_n. The vertex v_1 is connected to u_1 by an edge P_1, which collectively makes the path union of two copies of the cycle. Note that, |V(G)| = 2n and |E(G)| = 2n + 1.
We define labelling function $f: V(G) \rightarrow \{1,2,3,\dots,2n\}$ as follows.
Case 1: For $n \equiv 0,1,3(mod\ 4)$.
$f(v_i) = \{i\ ; i \equiv 0,1(mod 4)\ i+1; i \equiv 2(mod 4)\ i-1\ ; i \equiv 3(mod 4);\ 1 \leq i \leq n.$
Subcase 1: For $n \equiv 0(mod\ 4)$.

$f(u_i) = \{n+i\,; i \equiv 0,1(mod4)\; n+i+1\,; i \equiv 2(mod4)\; n+i-1\,; i \equiv 3(mod4);\; 1 \le i \le n.$

Subcase 2: For $n \equiv 1(mod\ 4)$.

$f(u_i) = \{n+i+1\,; i \equiv 1(mod4)\; n+i-1\,; i \equiv 2(mod4)\; n+i\,; i \equiv 0,3(mod4); 1 \le i \le n-1.$

$f(u_n) = 2n.$

Subcase 3: For $n \equiv 3(mod\ 4)$.

$f(u_i) = \{n+i+1\,; i \equiv 1(mod4)\; n+i-1; i \equiv 2(mod4)\; n+i\,; i \equiv 0,3(mod4); 1 \le i \le n.$

Case 2: For $n \equiv 2(mod\ 4)$.

$f(v_i) = \{i\,; i \equiv 0,1(mod4)\; i+1; i \equiv 2(mod4)\; i-1\,; i \equiv 3(mod4);\; 1 \le i \le n.$

$f(v_n) = n.$

$f(u_i) = \{n+i\,; i \equiv 0,1(mod4)\; n+i+1\,; i \equiv 2(mod4)\; n+i-1\,; i \equiv 3(mod4); 1 \le i \le n.$

$f(u_{n-1}) = 2n.$

$f(u_n) = 2n-1.$

In the above labelling pattern, we observe that,

Cases of n	Edge conditions
$n \equiv 1,3(mod\ 4)$	$e_f(0) = \lfloor \frac{2n+1}{2} \rfloor, e_f(1) = \lceil \frac{2n+1}{2} \rceil$
$n \equiv 0,2(mod\ 4)$	$e_f(1) = \lfloor \frac{2n+1}{2} \rfloor, e_f(0) = \lceil \frac{2n+1}{2} \rceil$

Clearly, $|e_{f(0)} - e_f(1)| \le 1$.

Hence, the path union of two copies of cycle C_n is a divided square difference cordial graph.

Example 1. Divided square difference cordial labeling of two copies of C_5 is shown in Figure 1.

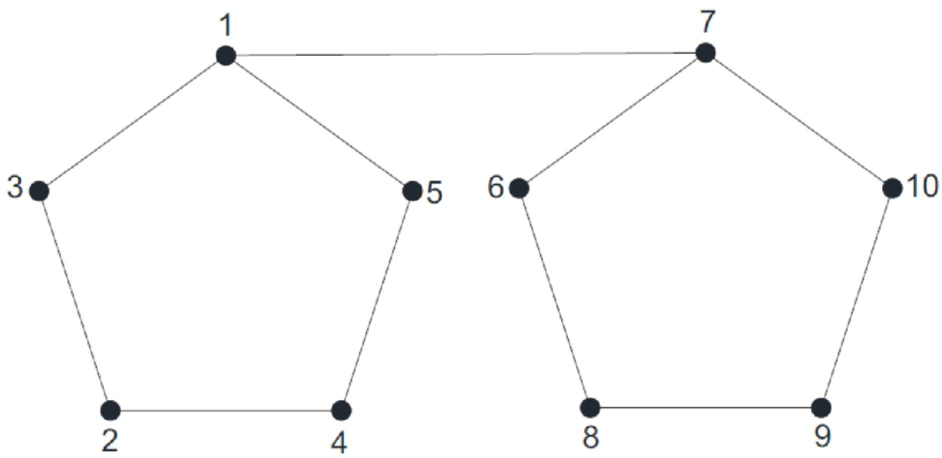

Figure 1

Theorem 2. The path union of two copies of cycle $C_{1,1n-2}$ $(n \in N, n \ge 5)$ with three chords is a divided square difference cordial graph, where three chords which by themselves forms a triangle.

Proof. Let G = (2. $C_{1,1,\,n-2}$) be the path union of two copies of cycle with three chords $C_{1,1n-2}$. Let $v_1, v_2, \dots, v_n$ be the successive vertices of C_n and let $u_1 = v_{n+1}, u_2 = v_{n+2}, \dots, u_n = v_{2n}$ be the successive vertices of another C_n. Let $e_1 = v_1v_3$ and $e_2 = v_3\, v_{n-1}$ and $e_3 = v_1v_{n-1}$be the chords of cycle C_n and let $e_1' = u_1\, u_3$ and $e_2' = u_3\, u_{n-1}$ and $e_3' = u_1\, u_{n-1}$ be the chords of cycle C_n. The vertex v_1 is connected to u_1 by an edge P_1, which collectively makes the path union of two copies of the cycle with three chords. Note that, |V(G)| = 2n and |E(G)| = 2n + 7.

We define labelling function $f: V(G) \to \{1,2,3, \dots, 2n\}$ as follows.

Case 1: For $n \equiv 0,2(mod\ 4)$.
$f(v_i) = \{i\ ; i \equiv 0,1(mod4)\ i+1\ ; i \equiv 2(mod4)\ i-1; i \equiv 3(mod4); 1 \leq i \leq n.$
Subcase 1: For $n \equiv 0(mod\ 4)$.
$f(u_i) = \{n+i\ \ ; i \equiv 0,1(mod4)\ n+i+1\ ; i \equiv 2(mod4)\ n+i-1\ ; i \equiv 3(mod4); 1 \leq i \leq n.$
Subcase 2: For $n \equiv 2(mod\ 4)$.
$f(u_i) = \{n+i-1\ \ ; i \equiv 1(mod4)\ n+i\ ; i \equiv 2,3(mod4)\ n+i+1\ ; i \equiv 0(mod4); 1 \leq i \leq n-1.$
$f(u_n) = 2n.$
Case 2: For $n \equiv 1(mod\ 4)$.
$f(v_i) = \{i\ ; i \equiv 0,1(mod4)\ i+1\ ; i \equiv 2(mod4)\ i-1\ ; i \equiv 3(mod4); 1 \leq i \leq n-1.$
$f(v_n) = n+1.$
$f(u_i) = \{n+i-1\ ; i \equiv 1(mod4)\ n+i\ ; i \equiv 2,3(mod4)\ n+i+1\ ; i \equiv 0(mod4)\ 1 \leq i \leq n.$
Case 3: For $n \equiv 3(mod\ 4)$.
$f(v_i) = \{i\ ; i \equiv 0,1(mod4)\ i+1\ ; i \equiv 2(mod4)\ i-1; i \equiv 3(mod4); 1 \leq i \leq n.$
$f(u_1) = n+1,$
$f(u_2) = n+2,$
$f(u_3) = n+3,$
$f(u_i) = \{n+i; i \equiv 0,3(mod4)\ n+i+1\ ; i \equiv 1(mod4)\ n+i-1; i \equiv 2(mod4); 4 \leq i \leq n.$
In the above labelling pattern, we observe that,

Cases of n	Edge conditions
$n \equiv 0,3(mod\ 4)$	$e_f(0) = \lfloor \frac{2n+7}{2} \rfloor, e_f(1) = \lceil \frac{2n+7}{2} \rceil$
$n \equiv 1,2(mod\ 4)$	$e_f(1) = \lfloor \frac{2n+7}{2} \rfloor, e_f(0) = \lceil \frac{2n+7}{2} \rceil$

Clearly, $|e_{f(0)} - e_f(1)| \leq 1.$
Hence, the path union of two copies of cycle with three chords $C_{1,1,n-2}$ is a divided square difference cordial graph.

Example 2. Divided square difference cordial labeling of two copies of $C_{1,1,n-2}$ is shown in Figure 2.

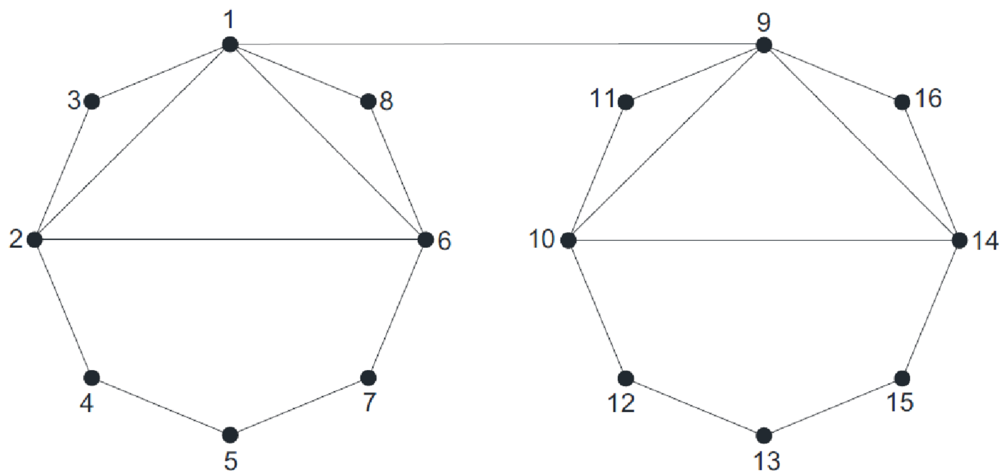

Figure 2

Theorem 3. The path union of two copies of wheel $W_n(n \epsilon N, n \geq 3)$ is a divided square difference cordial graph.
Proof. Let G = (2.W_n) be the path union of two copies of wheel W_n. Let $v_0, v_1, v_2, \dots, v_n$ be the successive vertices of W_n and let $u_0, u_1 = v_{n+1}, u_2 = v_{n+2}, \dots, u_n = v_{2n}$ be the successive vertices of another W_n. The vertex v_1 is connected to u_1 by an edge P_1 which collectively makes the path union of two copies of wheel. Note that, |V(G)|

= 2n + 2 and |E(G)| = 4n + 1.
We define labelling function $f: V(G) \to \{1,2,3, \dots ,2n + 2\}$ as follows.
$f(v_0) = 1$.
$f(v_i) = 2i + 2, 1 \leq i \leq n$.
$f(u_0) = 2$.
$f(u_i) = 2i + 1, 1 \leq i \leq n$.
In the above labeling pattern, observe that,

Cases of n	Edge conditions
$n \equiv 0,1,2,3(mod\ 4)$	$e_f(0) = \lfloor \frac{4n+1}{2} \rfloor, e_f(1) = \lceil \frac{4n+1}{2} \rceil$

Clearly, $|e_f(0) - e_f(1)| \leq 1$.
Hence, the path union of two copies of wheel W_n is a divided square difference cordial graph.

Example 3. Divided square difference cordial labelling of two copies of W_6 is shown in Figure 3.

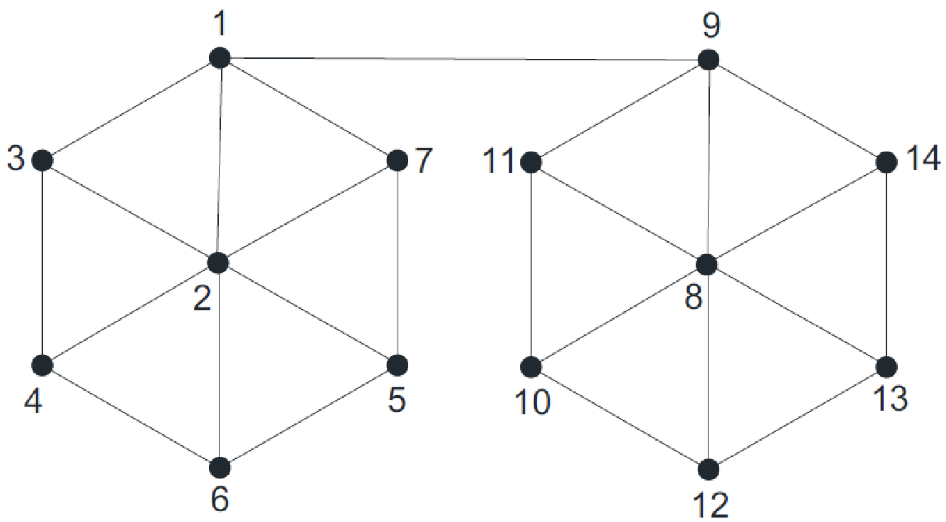

Figure 3

Theorem 4. The path union of two copies of helm H_n $(n \epsilon N, n \geq 3)$ is a divided square difference cordial graph.
Proof. Let G = (2.H_n) be the path union of two copies of helm H_n. Let $v_1, v_2, \dots, v_n$ be the vertices of degree 4 and $w_1, w_2, \dots, w_n$ be the pendant vertices, v_0 be apex vertex of H_n. Let $u_1 = v_{n+1}, u_2 = v_{n+2}, \dots, u_n = v_{2n}$ be the vertices of degree 4 and $w_1{}', w_2{}', \dots, w_n{}'$ be the pendant vertices, u_0 be apex vertex of another helm H_n. The vertex v_1 is connected to u_1 by an edge P_1 which collectively makes the path union of two copies of helm. Note that, |V(G)| = 4n + 2 and |E(G)| = 6n + 1.
We define labelling function $f: V(G) \to \{1,2,3, \dots ,4n + 2\}$ as follows.
Case 1: For $n \equiv 0,2(mod\ 4)$.
$f(v_0) = 1$,
$f(v_i) = \{2i + 1; i \equiv 1,3(mod4)\ 2i - 1; i \equiv 0,2(mod4);\ 1 \leq i \leq n$.
$f(w_i) = \{2i + 3;\ i \equiv 1,3(mod4)\ 2i; i \equiv 0,2(mod4);\ 1 \leq i \leq n$.
$f(u_0) = 2n + 2$,
$f(u_i) = \{2n + 2i + 2;\ i \equiv 1,3(mod4)\ 2n + 2i - 1;\ i \equiv 0,2(mod4);\ 1 \leq i \leq n$.
$f(w_i{}') = \{2n + 2i + 4;\ i \equiv 1,3(mod4)\ 2n + 2i + 1;\ i \equiv 0,2(mod4);\ 1 \leq i \leq n$.

Case 2: For $n \equiv 1,3(mod\ 4)$.
$f(v_0) = 1, f(v_n) = 2n + 1,$
$f(v_i) = \begin{cases} 2i + 1; i \equiv 1,3(mod4) \\ 2i - 1;\ \ i \equiv 0,2(mod4); \end{cases} 1 \le i \le n - 1.$
$f(w_n) = 2n$
$f(w_i) = \begin{cases} 2i + 3;\ i \equiv 1,3(mod4) \\ 2i;\ i \equiv 0,2(mod4); \end{cases} 1 \le i \le n - 1.$
$f(u_0) = 2n + 2, f(u_n) = 4n + 2$
$f(u_i) = \begin{cases} 2n + 2i + 2;\ \ i \equiv 1,3(mod4) \\ 2n + 2i - 1;\ i \equiv 0,2(mod4); \end{cases} 1 \le i \le n - 1.$
$f(w_n{}') = 4n + 1$
$f(w_i{}') = \begin{cases} 2n + 2i + 4;\ i \equiv 1,3(mod4) \\ 2n + 2i + 1;\ i \equiv 0,2(mod4); \end{cases} 1 \le i \le n - 1.$
In the above labeling pattern, observe that,

Cases of n	Edge conditions
$n \equiv 0,1,2,3(mod\ 4)$	$e_f(1) = \lfloor \frac{6n+1}{2} \rfloor, e_f(0) = \lceil \frac{6n+1}{2} \rceil$

Clearly,$|e_f(0) - e_f(1)| \le 1$.
Hence, the path union of two copies of helm H_n is a divided square difference cordial graph.
Example 4. Divided square difference cordial labelling of two copies of H_3 is shown in Figure 4.

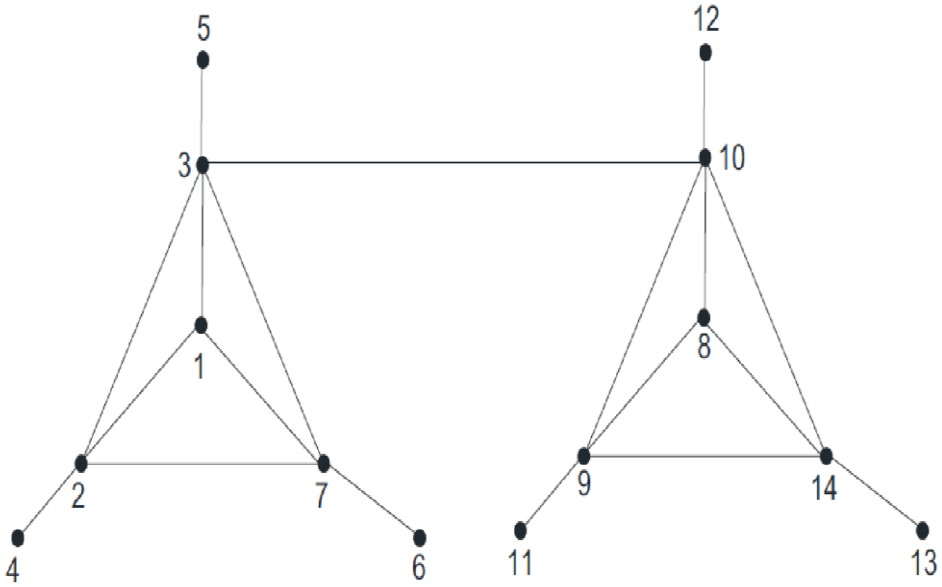

Figure 4

Theorem 5. The path union of two copies of closed helm $CH_n(n \epsilon N, n \ge 3)$ is a divided square difference cordial graph.
Proof. Let G = ($2.CH_n$) be the path union of two copies of closed helm CH_n. Let $v_1, v_2, \ldots, v_n$ be the internal vertices and $w_1, w_2, \ldots, w_n$ be the external vertices of CH_n. Let $u_1 = v_{n+1}, u_2 = v_{n+2}, \ldots, u_n = v_{2n}$ be the internal vertices and $w_1{}', w_2{}', \ldots, w_n{}'$ be the external vertices another closed helm CH_n. The vertex w_1 is connected to w_1' by an edge P_1 which collectively makes the path union of two copies of closed helm. Note that, |V(G)| = 4n + 2 and |E(G)| = 8n + 1.
We define labelling function $f: V(G) \to \{1,2,3, \ldots ,4n + 2\}$ as follows.
$f(v_0) = 1.$
$f(v_i) = 2i, 1 \le i \le n.$
$f(w_i) = 2i + 1, 1 \le i \le n.$
$f(u_0) = 2n + 2.$

$f(u_i) = 2n + 2i + 1, 1 \leq i \leq n.$
$f(w_i{}') = 2n + 2i + 2, 1 \leq i \leq n.$
In the above labeling pattern, observe that,

Cases of n	Edge conditions
$n \equiv 0,1,2,3(mod\ 4)$	$e_f(0) = \lfloor\frac{8n+1}{2}\rfloor, e_f(1) = \lceil\frac{8n+1}{2}\rceil$

Clearly, $|e_f(0) - e_f(1)| \leq 1.$
Hence, the path union of two copies of closed helm CH_n is a divided square difference cordial graph.

Example 5. Divided square difference cordial labelling of two copies of CH_7 is shown in Figure 5.

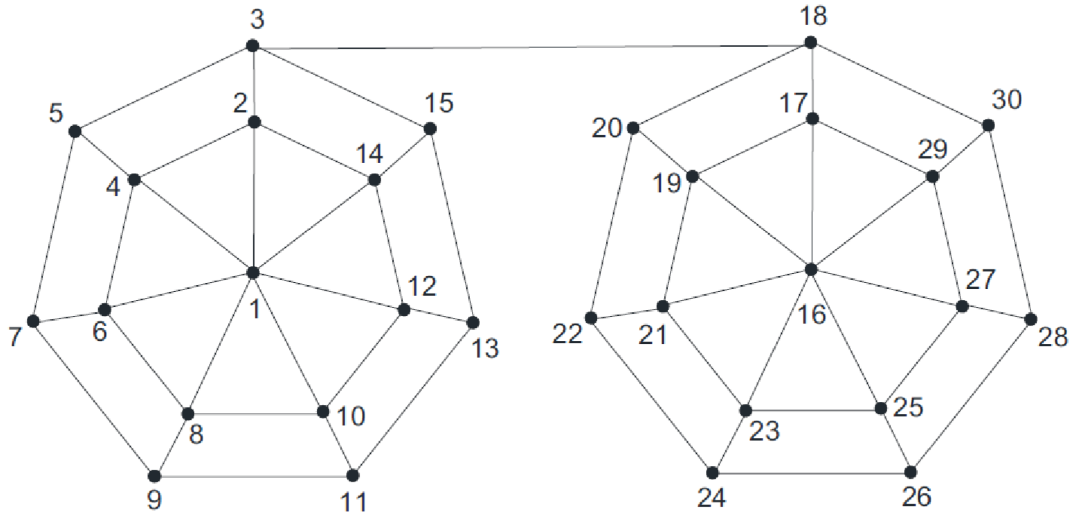

Figure 5

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