Anusandhanvallari
Vol 2022, No.1
August 2022
ISSN 2229-3388

A brief History of the Mathematical Development of Probability Theory

Abhisekh Shekhar*

*Assistant Professor Department of Mathematics C.M.Science College, Darbhanga-846004, Bihar, India (A constituent unit of L.N.Mithila University, Darbhanga-846008, Bihar, India)

Email Id: abhi08.iitkgp@gmail.com

Abstract: This article focuses on the evolution of the mathematical theory of probability, from its inception in a correspondence between Pascal and Fermat in 1654 to its peak in the early nineteenth century in Laplace's writings. It charts the evolution of the theory's applications and the meaning of mathematics during this time. Probability theory like many other branches of mathematics, evolved out of practical consideration.

Keywords: Probability, Game of chance, Statistics

2020 Mathematics Subject Classification: 97K50, 62-03, 01A05.

1|Introduction

In ancient times, Plato (428-348 BC) and his famous student, Aristotle (384-322 BC) used to discuss the word chance philosophically. In 324 BC, a Greek person, Antimenes (530-510 BC) first developed the system of insurance which guaranteed a sum of money against wins or losses of certain events. In view of many uncertainties of everyday life such as health, weather, birth, death and game that led to the concept of chance or random variables as output of an experiment (for example, the length of an object, the height of people, the temperature in a city in a given day). Almost all measurements in mathematics or science have the fundamental property that the results vary in different trials. In other words, results are, in general, random in nature. Thus, the quantity we want to measure is called a random variable. Historically, the word probability was associated with the Latin word 'probo' and the English words, probe and probable. In other languages, this word used in a mathematical sense had a meaning more or less like plausibility. In ancient times, the concept of probability arose in problems of gambling dealing with winning or losing of a game.

The earliest indication on measurement of chances in game of dice appeared in 1477 in a commentary on Dante's Divine Comedy. Since its inception, the study of probability has attracted the attention of great mathematicians

Fig.1.1: Robert Simpson Woodward (July 21, 1849-June 29, 1924)

Robert Simpson Woodward (July 21, 1849-June 29, 1924) was an American civil engineer, physicist and mathematician. R.S.Woodward told about probability theory is– **"The theory of probabilities and the theory of errors now constitute a formidable body of great mathematical interest and of great practical importance".**

Anusandhanvallari
Vol 2022, No.1
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Fig. 1.2: Charles Sanders Peirce (1839-1914)

Charles Sanders Peirce (1839-1914) was an educated as a chemist and employed as a scientist for thirty years, Peirce made major contributions to logic, such as theories of relations and quantification. C.S.Peirce said about probability **is – "The theory of probabilities is simply the Science of logic quantitatively treated".**

Although the idea of probability first appeared thousands of years ago, it wasn't until the middle of the seventeenth century that probability as a field of mathematics began to take shape. Even though mathematicians are still unfamiliar with the methods of calculation, the calculation of probabilities has become extremely evident in this period.

The earliest indication on measurement of chances in game of dice appeared in 1477 in a commentary on Dante's Divine Comedy. A treatise on gambling named liber de Ludo Alcae, by Geronimo Carden (1501-1576) was published posthumously in 1663. In this treatise, he gives the number of favourable cases for each event when two dice are thrown.

Galileo (1564-1642) gave casual remarks concerning the correct evaluation of chance in a game of three dice. Galileo analysed that when three dice are thrown, the sum of the number that appear is more likely to be 10 than the sum 9, because the number of cases favourable to 10 are more than the number of cases for the appearance of number 9.

Apart from these early contributions, it is generally acknowledged that the true origin of the science of probability lies in the correspondence between two great men of the seventeenth century, Pascal (1623-1662) and Pierre de Fermat (1601-1665). A French gambler, Chevalier de Metre asked Pascal to explain some seeming contradiction between his theoretical reasoning and the observation gathered from gambling. In a series of letters written around 1654, Pascal and Fermat laid the first foundation of science of probability. Pascal solved the problem in algebraic manner while Fermat used the method of combinations.

Great Dutch Scientist, Huygens (1629-1695), became acquainted with the content of the correspondence between Pascal and Fermat and published a first book on probability, "De Ratiociniis in Ludo Aleae" containing solution of many interesting rather than difficult problems on probability in games of chances.

The next great work on probability theory is by Jacob Bernoulli (1654-1705), in the form of a great book, "Ars Conjectendi" published posthumously in 1713 by his nephew, Nicholes Bernoulli. To him is due the discovery of one of the most important probability distribution known as Binomial distribution. The next remarkable work on probability lies in 1993. A. N. Kolmogorov (1903-1987) is credited with the axiomatic theory of probability. His book, 'Foundations of probability' published in 1933, introduces probability as a set function and is considered a 'classic!'.

In recent years, probability has been used extensively in many areas such as biology, economics, genetics, physics, sociology etc.

This manuscript discusses the journey of Probability from its inception i.e. the classical approach to the applied probability in the strong mathematical sense i.e. the Measure Theory. We also dwelt upon the illustrations of how these theories were chiseled and forged into what we see today. Though it arose out of the study of gambling games but it has a range of applications extends beyond games into business decisions, insurance, law, medical tests, and the social sciences. This paper provides an informative perception towards the History and Development of Probability.

2 | The Mathematical literature of Probability

Probability has been developed by the contribution of many scientists and mathematicians, it is discussed in this section.

2.1 | Indian aspects of Probability in the Mahabharata

Fig.2.1: Gambling in Mahabharta

Probability theory in the Mahabharata is primarily illustrated through the game of dice (dyuta) between Yudhishthira and Shakuni, which functions as a metaphor for fate, risk, and loaded probability. It explores the distinction between fair gambling and, through Shakuni's magical dice, rigged, zero-sum games where the probability of the Pandavas winning was effectively zero.

2.2 | Gerolamo Cardano

Fig.2.2 Gerolamo Cardano

Probability theory had its origin in the 16th century when an Italian physician and mathematician G.Cardano (1501–1576) wrote the first book on the subject, "The Book on Games of Chance" (Biber de Ludo Aleae). It was published in 1663 after his death, contains the first systematic treatment of probability as well as a section on effective cheating methods. He used the game of throwing dice to understand the basic concepts of probability. He demonstrated the efficacy of defining odds as the ratio of favourable to unfavourable outcomes (which implies that the probability of an event is given by the ratio of favourable outcomes to the total number of possible outcomes). He was also aware of the multiplication rule for independent events but was not certain about what values should be multiplied. In this treatise, he gives the number of favourable cases for each event when two dice are thrown.

2.3 | Pierre Simon Laplace

Fig.2.2: Pierre Simon Laplace

Pierre-Simon Laplace (1749–1827), often called the "Newton of France," fundamentally transformed probability theory from studying games of chance into a vital analytical tool for science. He defined probability as the ratio of favorable cases to total possible cases, pioneered **Bayesian interpretation** (inverse probability), and developed the first central limit theorem. Pierre Simon Laplace is among those who made significant contributions to this field. Laplace's Theorie Analytique des Probabilités, 1812, is considered to be the greatest contribution by a single person to the theory of probability.

Key contributions by Laplace to probability include:

Analytical Probability Theory: In his work " Théorie analytique des probabilities""(1812), Laplace developed tools to analyze probability, moving beyond basic combinatorics.

Classical Definition: He formalized the classical definition of probability: the number of favorable cases divided by the total number of equally possible cases.

Inverse Probability (Bayesian): Laplace significantly advanced what is now known as Bayesian probability, using observed data to determine the probability of causes (inverse probability).

Central Limit Theorem: He proved an early form of the Central Limit Theorem, showing that the sum of independent random variables tends toward a normal distribution.

Laplace Distribution : The Laplace distribution, or double exponential distribution, is named after him, which is often used in modeling, including in scenarios like modeling errors.

Applications: He extended probability to real-world areas beyond games of chance, such as decision-making, population statistics, and mortality tables.

Scientific Determinism: While a pioneer of probability, he also famously proposed an intellectual "demon" that could predict the future with certainty if it knew all forces and positions in the universe, representing a pinnacle of classical deterministic thought.

Laplace's work established the foundation for modern probability theory, transforming it into a vital tool for science and statistics.

2.4 | Galileo di Vincenzo Bonaiuti de' Galilei

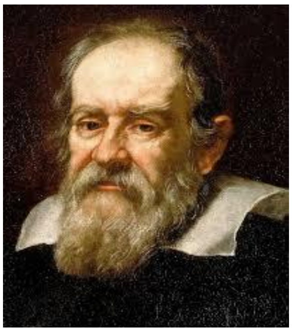

Fig. 2.4: Galileo

He commonly referred to as **Galileo Galilei**, also mononymously as **Galileo**, was an Italian astronomer, physicist, and engineer, sometimes described as a polymath. Galileo has been called the father of observational astronomy, modern-era classical physics, the scientific method, and modern science.

Galileo gave casual remarks concerning the correct evaluation of chance in a game of three dice. Galileo analysed that when three dice are thrown, the sum of the number that appear is more likely to be 10 than the sum 9, because the number of cases favourable to 10 are more than the number of cases for the appearance of number 9.

2.5 | Blaise Pascal and Pierre de Fermat

Fig.2.5.1: Blaise Pascal and Pierre de Fermat

Apart from these early contributions, it is generally acknowledged that the true origin of the science of probability lies in the correspondence between two great men of the seventeenth century, Pascal (1623-1662) and Pierre de Fermat (1601-1665). A French gambler, Chevalier de Metre asked Pascal to explain some seeming contradiction between his theoretical reasoning and the observation gathered from gambling. In a series of letters written around 1654, Pascal and Fermat laid the first foundation of science of probability. Pascal solved the problem in algebraic manner while Fermat used the method of combinations**.**

Blaise Pascal founded modern probability theory in 1654 alongside Pierre de Fermat, primarily to solve gambling problems like the “problem of points". By analyzing, for example, how to fairly split stakes in an unfinished game, he developed concepts of expected value, combinations, and “Pascal’s triangle”, shifting the view of uncertainty toward mathematical analysis.

Key aspects of Pascal's contributions to probability include:

The Problem of Points: Addressed how to divide the stakes in a game of chance if it is interrupted before completion, based on the probability of each player winning from that point.

Expected Value: Introduced the concept of calculating the expected return of a game to determine fair, advantageous, or disadvantageous situations.

Pascal's Triangle: Utilized his pre-existing work on combinations and arithmetic triangles to determine the probability of specific outcomes in games.

Dependency of Events: Recognized that the probability of future events can depend on previous ones, laying foundational probabilistic logic.

Pascal's Wager: Applied probabilistic reasoning to philosophy, arguing that believing in God is the most rational choice due to the "infinite" payoff of being right versus the finite loss of being wrong.

The collaboration with Fermat on these problems arose from questions posed by the Chevalier de Méré regarding dice games, which transformed chance into a rigorous mathematical field.

2.6 | Christiaan Huygens

Fig.2.6: Christiaan Huygens

Great Dutch Scientist, Huygens, became acquainted with the content of the correspondence between Pascal and Fermat and published a first book on probability, "De Ratiociniis in Ludo Aleae" containing solution of many interesting rather than difficult problems on probability in games of chances.

2.7 | Jacob Bernoulli

Fig.2.7: Jacob Bernoulli

The next great work on probability theory is by Jacob Bernoulli (1654-1705), in the form of a great book, "Ars Conjectendi" published posthumously in 1713 by his nephew, Nicholes Bernoulli, eight years after his death. The work was incomplete at the time of his death but it is still a work of the greatest significance in the theory of probability. The book also covers other related subjects, including a review of combinatirics, in particular the work of van Schooten, Leibniz, and Prestet, as well as the use of Bernoulli numbers in a discussion of the exponential series. Inspired by Huygens' work, Bernoulli also gives many examples on how much one would expect to win playing various games of chance. The term Bernouli trials resulted from this work.

In the last part of the book, Bernoulli sketches many areas of mathematical probability, including probability as a measurable degree of certainty; necessity and chance; moral versus mathematical expectation; a priori and a

Anusandhanvallari
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posteriori probability; expectation of winning when players are divided according to dexterity; regard of all available arguments, their valuation, and their calculable evaluation; and the law of large numbers.

To him is due the discovery of one of the most important probability distribution known as Binomial distribution.

2.8 | Andrei Nikolaevich Kolmogorov

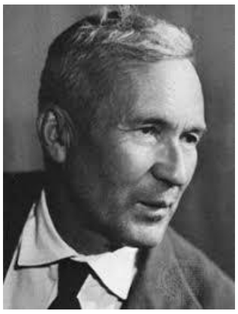

Fig.2.8: A. N. Kolmogorov Andrey Andrei Nikolaevich Kolmogorov (1903-1987) was a Soviet mathematician who played a central role in the creation of modern probability theory. A. N. Kolmogorov is credited with the axiomatic theory of probability. His book, 'Foundations of probability' published in 1933, introduces probability as a set function and is considered a 'classic!'.

3 | Probability — A Theoretical Approach

Fig.3.1: Tossing a coin

Let us consider the following situation: Suppose a coin is tossed at random. We know, in advance, that the coin can only land in one of two possible ways —either head up or tail up (we dismiss the possibility of its 'landing' on its edge, which may be possible, for example, if it falls on sand). We can reasonably assume that each outcome, head or tail, is as likely to occur as the other. We refer to this by saying that the outcomes head and tail are equally likely.

For another example of equally likely outcomes, suppose we throw a die once. For us, a die will always mean a fair die. What are the possible outcomes? They are 1, 2, 3, 4, 5, 6. Each number has the same possibility of showing up. So the equally likely outcomes of throwing a die are 1, 2, 3, 4, 5 and 6.Are the outcomes of every experiment equally likely? Let us see. Suppose that a bag contains 4 red balls and 1 blue ball, and you draw a ball without looking into the bag. What are the outcomes? Are the outcomes—a red ball and a blue ball equally likely? Since there are 4 red balls and only one blue ball, you would agree that you are more likely to get a red ball than a blue ball. So, the outcomes (a red ball or a blue ball) are not equally likely. However, the outcome of drawing a ball of any colour from the bag is equally likely. So, all experiments do not necessarily have equally likely outcomes. However, in this section, from now on, we will assume that all the experiments have equally likely outcomes.

We discuss the experimental or empirical probability P (E) of an event E as

$$P(E) = \frac{Number\ of\ trials\ in\ which\ the\ event\ happened}{Total\ number\ of\ trials}$$

The empirical interpretation of probability can be applied to every event associated with an experiment which can be repeated a large number of times. The requirement of repeating an experiment has some limitations, as it may be very expensive or unfeasible in many situations. Of course, it worked well in coin tossing or die throwing experiments. But how about repeating the experiment of launching a satellite in order to compute the empirical probability of its failure during launching, or the repetition of the phenomenon of an earthquake to compute the empirical probability of a multi-storeyed building getting destroyed in an earthquake? In experiments where we are prepared to make certain assumptions, the repetition of an experiment can be avoided, as the assumptions help in directly calculating the exact (theoretical) probability. The assumption of equally likely outcomes (which is valid in many experiments, as in the two examples above, of a coin and of a die) is one such assumption that leads us to the following definition of probability of an event.

The theoretical probability (also called classical probability) of an event E,

written as P(E), is defined as

$$P(E) = \frac{Number\ of\ outcomes\ favourable\ to\ E}{Number\ of\ all\ possible\ outcomes\ of\ the\ experiment}$$

where we assume that the outcomes of the experiment are equally likely. We will briefly refer to theoretical probability as probability.

This definition of probability was given by Pierre Simon Laplace in 1795

4 |Declarations

Funding. The author received no financial support for the research, authorship, or publication of this article.

Conflict of Interest. The author declares that there are no conflicts of interest regarding the publication of this manuscript.

Ethical Approval. This article does not contain any studies with human participants or animals performed by the author.

Data Availability. No datasets were generated or analyzed during the current study.

Author Contributions. The author carried out the entire research work independently, including the formulation of the problem, development of the theoretical framework, derivation of the mathematical results, preparation of proofs, numerical computation, and writing of the manuscript.

Acknowledgements. The author would like to express sincere gratitude to the faculty and research environment of C.M.Science College, Darbhanga-846004, Bihar, India for continuous academic support. The author is grateful for the useful critiques and suggestions of the reviewers. I also thank reviewers for their helpful suggestions, insightful comments, and valuable feedback. The author would also like to thank the editor of the journal for his constructive suggestions.

References

[1] Ore, O. Pascal and the invention of probability theory. *The American Mathematical Monthly*, 1960, *67*(5), 409-419.

[2] Kolmogorov, Andrey (1950) [1933].Foundations of the theory of probability. New York, US: Chelsea Publishing Company.

[3] Daston, Lorraine. Classical Probability in the Enlightenment. Princeton (Princeton University Press).1988.

[4] Hald, Anders 1990. A History of Probability and Statistics and their Applications before. New York (Wiley).1750.

[5] Kendall, Maurice G., and R. L. Plackett, eds. Studies in the History of Statistics and Probability. Volume 2. New York (Macmillan).1977.
[6] Todhunter, Isaac. A History of the Mathematical Theory of Probability. London (Macmillan). 1865
[7] Edwards, A.W.F. Pascal's Arithmetic Triangle. London (Griffin).1987.
[8] Dr. young Xu, History, Background and probability applications,2012.
[9] Debnath, Lokenath, and Kanadpriya Basu. "A short history of probability theory and its applications." *International Journal of Mathematical Education in Science and Technology* 46.1 (2015): 13-39.